Neutrinoless Double Beta Decay

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Abstract

I give a review of the theory and some of the experiments pertaining to neutrinoless double beta decay ($0\nu\beta\beta$). In certain atoms, it is favorable to undergo a beta decay with $\Delta Z = 2$ and emission of two electrons. If the neutrino is not charged under any complex representation of a gauge group, it should be possible to observe such a double beta decay without neutrinos. In order to study this phenomenon, I will first briefly review the QFT of Majorana fermions. I then review the calculation of the leptonic part of the $0\nu\beta\beta$. I will proceed to briefly discuss the relevant nuclear and atomic physics. I also give an overview of experiments designed to observe $0\nu\beta\beta$.

I. MAJORANA FERMIONS

I begin this review with a review of a basic quantum field theory topic that rarely gets covered in QFT classes. Neutrinoless double beta decay ($0\nu\beta\beta$) inherently depends on the presence of a Majorana mass term for the neutrino. If the neutrino is in fact a Dirac fermion, the amplitude for $0\nu\beta\beta$ would vanish identically. Therefore, I will discuss Majorana fermions and briefly review the application of Feynman rules in both the two-component and four-component notations.

I will define a Majorana fermion as one for which it is possible to write down a Majorana mass term[1]. A theory having only a single Majorana fermion would have the Lagrangian

$$\mathcal{L} = i\chi_{\dot{\alpha}}\overline{\sigma}^{\dot{\alpha}\alpha} \cdot \partial\chi_\alpha - \frac{m}{2}(\chi^\alpha\chi_\alpha + \chi^{\dagger\dot{\alpha}}\chi^\dagger_{\dot{\alpha}}), \tag{1}$$

where χ is a two-component Weyl fermion. This can be written a four component notation as

$$\mathcal{L} = \frac{i}{2}\overline{\Psi}\partial\!\!\!/\Psi - \frac{m}{2}\overline{\Psi}\Psi, \tag{2}$$

where we define by hand $\Psi = (\chi_\alpha \quad \chi^{\dagger\dot{\alpha}})^T$. However, for the purposes of writing down Feynman rules and understanding symmetries, I think it is easier to write this Lagrangian in terms of a usual Dirac fermion Ψ and to use projection and charge conjugation operators:

$$\mathcal{L} = i\overline{\Psi_L}\partial\!\!\!/\Psi_L - \frac{m}{2}(\overline{\Psi^C_R}\Psi_L + \overline{\Psi_L}\Psi^C_R). \tag{3}$$

The quantity Ψ^C_R is defined by

$$\Psi^C_R = -iP_R\gamma^2\Psi^*. \tag{4}$$

This is just an explicit way to write out equation (1) in terms of a general Dirac fermion. Notice that Ψ^C_R has the conjugate quantum numbers of Ψ_L. Using either this version of the four-component Lagrangian or the two-component Lagrangian, it is trivial to see that if the fermion is charged under a $U(1)$ or $SU(N)$, the mass term written is forbidden. Conversely, if a fermion is not charged under complex groups, one might expect a Majorana mass term to be present. This statement is not exactly true, as an accidental chiral symmetry will still

[1] Some authors will define a Majorana fermion as having a real representation under the Dirac algebra. For our purposes, my definition will be more useful. Another common definition is that a Majorana fermion is one that is its own antiparticle. Both of these definitions are equivalent to mine.

protect against such a term. The mass term written breaks the $U(1)$ global symmetry of the kinetic term. Chiral symmetry not withstanding, neutral fermions will generically have Majorana mass terms.

Consider an example to see how such a term might arise. In the Standard Model, the neutrino component of the left-handed lepton doublet is neutral under all unbroken complex symmetries ($U(1)_{\text{EM}}$ and $SU(3)_{\text{C}}$). Furthermore, we know that the Higgs will break chiral symmetry. Therefore, we should be able to write down a Majorana mass term. Indeed the one and only dimension 5 operator in the SM generates a Majorana mass for the neutrino:

$$\mathcal{L}_5 = \frac{\lambda_{ij}}{\Lambda}(H^T \ell_i^\alpha)(H^T \ell_{j\alpha}) + \text{h.c.} = \frac{\lambda_{ij}}{\Lambda}(\overline{L_{iR}^C} H)(H^T L_{jL}) + \text{h.c.}. \tag{5}$$

When the Higgs gets a VEV $\langle h \rangle = v/\sqrt{2}$, the neutrinos get a Majorana mass matrix

$$(m_\nu)_{ij} = \frac{\lambda_{ij} v^2}{\Lambda}. \tag{6}$$

The amplitude for $0\nu\beta\beta$ is proportional to this mass matrix, as I will justify further in the next section.

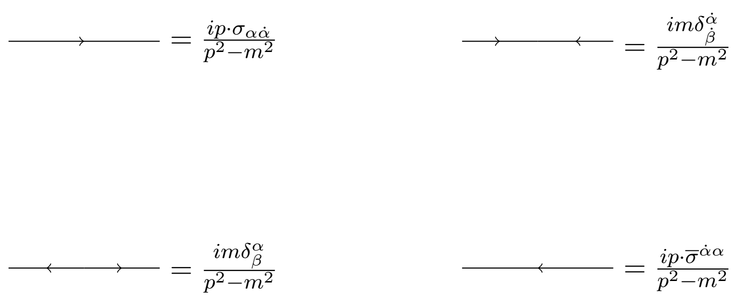

FIG. 1. Feynman rules for Majorana fermions in the two-component notation.

For now, I want to quickly outline how to work with Feynman rules and Majorana fermions, since this is often a point of much confusion. I find it easiest to work in the two component notation. The review by Dreiner et. al. outlines how to deal with Majorana neutrinos very nicely. It is actually no different than dealing with Dirac neutrinos, except for the fact that there is one less (two-component) fermion to deal with! Still, for reference, I write out the two component Feynman rules in Figure 1.

In the four-component notation, there is more room for confusion. It is easiest to treat all vertices as normal for the four component notation (that is, vertices do not change the

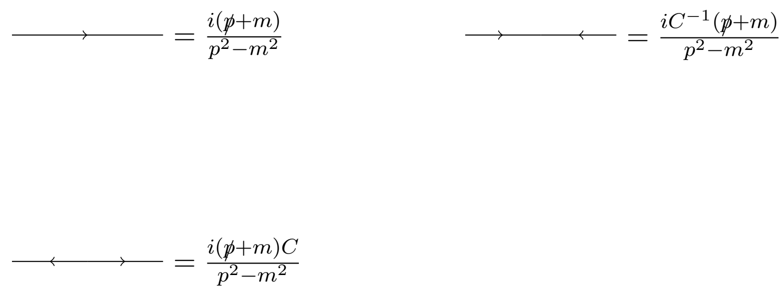

FIG. 2. Feynman rules for Majorana fermions in the four-component notation. In this notation, $C = -i\gamma^2\gamma^0$ and $C^{-1} = i\gamma^2\gamma^0$.

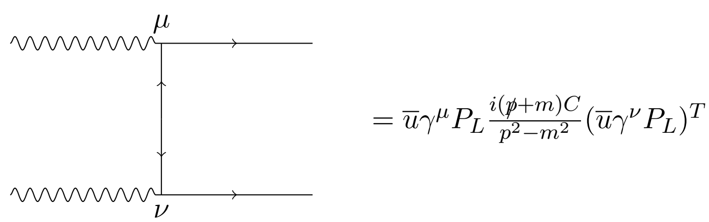

FIG. 3. Example of a diagram using the four-component notation. This diagram represents the Dirac space structure of the diagam for $0\nu\beta\beta$.

direction of fermion lines). Then, the only modification is to the Majorana fermion propagator. Also, it is now possible two inward-pointing or outward-pointing external fermions that are part of the same line. To deal with this situation, it is necessary to take a transpose of one half of the line. See the Figure for an example of how this is done.

II. NEUTRINO PHENOMENOLOGY AND THE PARTICLE PHYSICS AMPLITUDE

With the formal QFT understood, we now move on to discuss neutrino phenomenology. Our starting point is the lepton sector Lagrangian of the standard model, including the dimension 5 operator discussed above:

$$\mathcal{L} = i\ell_i^\dagger \overline{\sigma} \cdot D\ell_i + ie_i^{C\dagger}\overline{\sigma} \cdot De_i^C - Y_{ij}^\ell \ell_i H e_j^C - \frac{\lambda_{ij}}{\Lambda}(\ell_i H)(\ell_j H) + \text{h.c.}. \tag{7}$$

The kinetic term has a $U(3)^2$ symmetry, which is broken completely. Thus, there are 6 real and 12 imaginary unphysical parameters. The matrix Y^ℓ is arbitrary and complex, so it has 9 real and 9 imaginary parameters. The matrix λ is complex and symmetric since it couples

ℓ to ℓ, so it has 6 real parameters and 6 phases. That means that there are 9 physical real parameters (3 charged lepton masses, 3 neutrino masses, 3 mixing angles) and 3 physical imaginary phases (1 Dirac phase and 2 Majorana phases). After electroweak symmetry breaking and flavor rotation, the W interaction terms are

$$\mathcal{L}_{\text{int}} = -\frac{g}{\sqrt{2}}\left[(U^*_{\text{PMNS}})_{ij}\nu^\dagger_j\overline{\sigma}\cdot W^+\ell_i + (U_{\text{PMNS}})_{ij}\ell^\dagger_i\overline{\sigma}\cdot W^-\nu_j\right], \tag{8}$$

where U_{PMNS} is the Pontecorvo-Maki-Nakagawa-Sakata lepton mixing matrix. I will abbreviate $U_{\text{PMNS}} = U$ from now on. One can choose a basis such that

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}\begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{9}$$

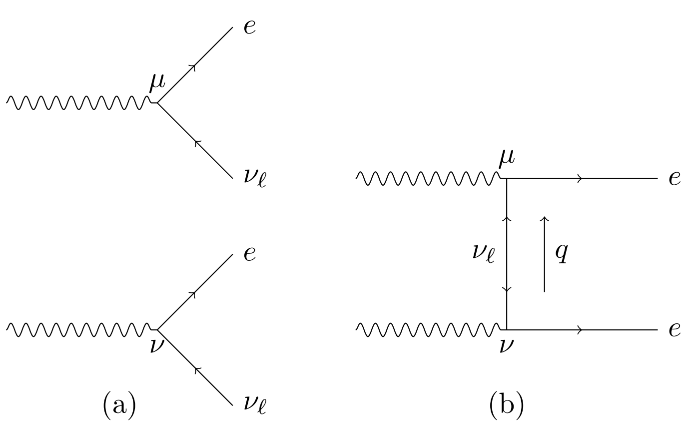

FIG. 4. (a) Diagram for double beta decay. (b) Diagram for neutrinoless double beta decay.

That is all the theory necessary to understand $0\nu\beta\beta$. In double beta decay, we encounter a diagram as shown in Figure 4(a). If the neutrino is a Majorana neutrino, then we can connect the two external neutrino legs to form Figure 4(b). This is the diagram for $0\nu\beta\beta$. Let's calculate this diagram. Using the two-component notation,

$$i\mathcal{M}^{\mu\nu} = x^\dagger\left(i\frac{g}{\sqrt{2}}\right)\overline{\sigma}^\mu U_{1j}\frac{im_j}{q^2 - m_j^2}\left(i\frac{g}{\sqrt{2}}\right)\sigma^\nu U_{1j}x^\dagger. \tag{10}$$

In principle, we can calculate this part of the diagram. However, for now, let's focus on the neutrino parameter dependence of this diagram. From this amplitude, we can see that the rate for $0\nu\beta\beta$ must be proportional to

$$\Gamma \propto |\sum_j U_{1j}^2 m_j|^2. \tag{11}$$

This rate is dependent on the CP violating Majorana phases α_1 and α_2. However, in order to actually determine these phases, it is necessary to first determine all three neutrino masses. The only quantity that can be determined from $0\nu\beta\beta$ is

$$\langle m \rangle \equiv |\sum_j U_{1j}^2 m_j|. \tag{12}$$

However, determining this quantity does but a fairly tight bound on the absolute neutrino mass scale even if it cannot be determined through precision kinematics of tritium beta decay. Note that

$$\max(2|U_{1j}|^2 m_j - \sum_j |U_{1j}|^2 m_j, 0) < \langle m \rangle < \sum_j |U_{1j}|^2 m_j. \tag{13}$$

By measuring $\langle m \rangle$ as well as all the mixing angles and mass splittings, we get a range for the undetermined mass. This range does depend on whether we choose a normal or inverted mass scheme.

Neutrinoless double beta decay is not the only process that could grant us access to the mass scale and Majorana phases. There is the previously mentioned tritium beta decay experiment. By making precision measurements of the spectrum of electrons coming out of the decay, it may be possible to determine the mass of neutrinos coming out of the decay. Neutrino mass should lead to an energy cutoff in the spectrum slightly below that expected in the absence of neutrino mass. Unfortunately, decays near the cutoff of the spectrum are extremely rare and so measurement of the neutrino mass via this mechanism is extremely difficult.

In addition, there are other lepton number-violating processes we could look for. One idea is to fire muons at atoms and look for conversion to positrons. There should also be kaon decays to like-sign leptons. Unfortunately, we do not have the precision required to observe the Majorana phase through these processes.

III. NUCLEAR MATRIX ELEMENTS

Perhaps the most difficult part of studying $0\nu\beta\beta$ is calculating with sufficient accuracy the nuclear matrix element involved in the decay. For our purposes, it is important to understand that it is very difficult to calculate these matrix elements. However, I will go into some detail as to exactly what is involved. Be forewarned that I know little about nuclear physics and some facts may be taken verbatim from the review. The calculations involve various approximation schemes for the nuclear wavefunction. The two most popular schemes are the quasiparticle random phase approximation (QRPA) and the nuclear shell model (NSM). My understanding is that the QRPA is less reliable, but easier computationally. Unfortunately, these elements depend intrinsically on the fact that the process is neutrinoless: there is no reliable way to compare with the usual β decay matrix elements. Therefore, estimates of the uncertainty on the nuclear matrix element calculations are not reliable.

The process we are considering is some nucleus of mass M_i and atomic/mass numbers (Z, A) going to a final state with a nucleus of mass M_f, energy E_f, and atomic/mass numbers $(Z+2, A)$ with the emission of electrons of energy $\epsilon_{1,2}$. From quantum mechanics, we know that we can write the differential decay rate as

$$d\Gamma = \sum_{\text{spin}} |R_{0\nu}|^2 (2\pi)^4 \delta^{(4)}(p_i - p_f - p_1 - p_2) \frac{d^3 p_f}{(2\pi)^3} \frac{d^3 p_1}{(2\pi)^3} \frac{d^3 p_1}{(2\pi)^3}. \tag{14}$$

It is useful (or maybe just conventional) to write out the amplitude (10) in position space and to integrate over the four-momentum of the virtual neutrino. The result has the form

$$\mathcal{M}_{\mu\nu}(r) \propto \frac{m_j U_{1j}^2}{\omega_j} \frac{H_1(r, A_1) + H_2(r, A_2)}{2} \tag{15}$$

where r is the distance between the neutrons that are being converted into protons, H_k is the neutrino weak potential, and A_k is a factor dependent on the excitation energy E_m of the intermediate nuclear state defined by

$$A_{1(2)} = E_m - \frac{M_i + M_f}{2} \pm \frac{\epsilon_1 - \epsilon_2}{2}. \tag{16}$$

A couple of unjustified facts are that the average of the two potentials is roughly given by the potential evaluated for for the average A:

$$\frac{H_1(r, A_1) + H_2(r, A_2)}{2} \approx H(r, \overline{A}), \tag{17}$$

and that the potential is essentially independent of the neutrino mass so long as the neutrino is lighter than about 10 MeV. Putting all these pieces together, we get a final master formula for the $0\nu\beta\beta$ decay rate that has factored into three pieces:

$$\Gamma = G^{0\nu}(E_0, Z)|M^{0\nu}|^2 \langle m_\nu \rangle^2. \tag{18}$$

$G^{0\nu}$ is a phase-space factor

$$G^{0\nu}(E_0, Z) \sim \int F(Z, \epsilon_1)F(Z, \epsilon_2)p_1p_2\epsilon_1\epsilon_2\delta(E_0 - \epsilon_1 - \epsilon_2)d\epsilon_1 d\epsilon_2, \tag{19}$$

where $F(Z, E)$ is the Fermi function which accounts for the fact that protons in the nucleus will decelerate the electrons and E_0 is the energy available to the electrons. $|M^{0\nu}|$ is the problematic nuclear matrix element and is given by

$$|M^{0\nu}| = M^{0\nu}_{\mathrm{GT}} - \frac{g_V^2}{g_A^2}M^{0\nu}_F = \langle f| \sum_{\ell k} H(r_{\ell k}, \overline{A})\tau_\ell^+\tau_k^+ \left(\vec{\sigma}_\ell \cdot \vec{\sigma}_k - \frac{g_V^2}{g_A^2}\right)|i\rangle, \tag{20}$$

where $M^{0\nu}_{\mathrm{GT}}$ and $M^{0\nu}_{\mathrm{F}}$ are called the Gamow-Teller and Fermi matrix elements respectively, i (f) describe the nuclear initial and final states, which are taken to be ground states of their respective nuclei, τ^+ are isospin raising operators, and $\vec{\sigma}$ are spin angular momentum Pauli matrices. The phase space factors can be evaluated accurately. The difficult part is evaluating $M^{0\nu}_{\mathrm{GT}}$ and $M^{0\nu}_{\mathrm{F}}$. They are likely to depend on poorly known pieces of the nuclear wavefunction.

The calculations of the lifetime appear to be known only to within an order of magnitude or so. I can do about that well! Let's estimate the lifetime of a nucleus undergoing $0\nu\beta\beta$. Clearly, the rate must be proportional to $G_F^4 m_\nu^2$. The remaining factors must come from nuclear physics. The phase space for this is highly suppressed since $\delta m \equiv m_p - m_n \ll m_n, m_p$. Looking at the phase space integral, there are six powers of the electron energies involved. The electron energies are roughly δm. So, we expect six powers of δm. The remaining power should then come from the nuclear matrix element. Since we have accounted for the phase space, neutrino and electroweak scales, the only scale left is that of the nucleons, m_p. So we can estimate that

$$\Gamma \sim m_p(\delta m)^6 G_F^4 m_\nu^2 \approx \left(\frac{m_\nu}{50\ \mathrm{meV}}\right)^2 \frac{1}{10^{26}\ \mathrm{years}}, \tag{21}$$

which matches the order of magnitude of the calculated matrix elements. The calculations show half-lives from 10^{25} to 10^{28} years depending on the isotope and calculation.

IV. EXPERIMENTAL SEARCHES

In this section, I will give an overview of the experimental techniques and searches for $0\nu\beta\beta$. The idea of all experimental searches is simple. A sample of an isotope for which double beta decay dominates (because of either kinematics or symmetry) is placed near a calorimetric detector. Then, one looks for coincident electrons, measures their energies, and sums them. The sum of the electron energies in the case of $0\nu\beta\beta$ is sharply peaked at some value set by the mass difference between the initial and final isotope masses. In the case with neutrinos (or other neutral particles), this energy follows a distribution. By applying a cut on the energies, one hopes to see a peak that rises above any backgrounds.

This approach dictates the criteria set out by the experiments. Strong energy resolution is required in order to focus as narrowly on the peak as possible. Relatively high energy decays are advantageous since they generally have a higher rate and less background. The lifetime for 0ν decay is generally several orders of magnitude larger than the 2ν lifetime, so a large source and good efficiency are necessary to increase sensitivity. Finally, backgrounds from other radioactive processes are a major concern, so one would like to minimize the amount of radioactive contaminants in the source and detector and the cosmic ray background.

Experiment	Isotope	$\langle m_\nu \rangle$ (eV)	Status
Heidelberg-Moscow	^{76}Ge	0.39 (detection)	Complete
IGEX	^{76}Ge	0.33 – 1.35	Complete
CUORICINO	^{130}Te	0.19 – 0.68	Running
NEMO	^{100}Mo and ^{82}Se	4.0 – 6.3	Running
EXO-200	^{136}Xe	0.2 – 0.7	Construction
GERDA	^{76}Ge	0.07 – 0.2	Funded
MAJORANA	^{76}Ge	0.1 – 0.3	Funded
CANDLES	^{48}Ca	0.5	Funded
SuperNEMO	^{82}Se	0.07 – 0.12	R&D
SNO++	^{150}Nd	0.03	R&D
DCBA	^{150}Nd	0.07	R&D

TABLE I. Results and expected limits of some $0\nu\beta\beta$ experiments.

All of the current and proposed experiments fall into one of several categories. One of the most successful types of experiments so far uses germanium as both the source and detector, taking advantage of its natural contamination by an isotope that undergoes double beta decay. Examples of experiments that use this technique are Heidelberg-Moscow, IGEX, GERDA, and MAJORANA. Heidelberg-Moscow actually claims a 4σ signal, but the claim is highly controversial. Alternatives to this technique include xenon gas detectors (also using the detector as the source), thermal detectors (measuring energy deposits by looking for increases in temperature), liquid detectors, scintillators, and stacks of silicon wafers. Several experiments and their sensitivities are summarized in Table I.

V. REFERENCES

I will not cite the source for each particular formula. Instead, I summarize the sources I used for each section. For section I, my primary reference for using the two component notation was [1]. For section II, I referred to [2] for the low energy effective theory of neutrinos. For section III, I referred primarily to [3, 4] and sources therein. For section IV, I referred to [3–6]. In particular, I would like to highlight [6] as a great resource for finding the most important papers in the field, sorted by experiment.

[1] H. K. Dreiner, H. E. Haber and S. P. Martin, arXiv:0812.1594 [hep-ph].

[2] Y. Grossman, arXiv:hep-ph/0305245.

[3] S. R. Elliott and P. Vogel, Ann. Rev. Nucl. Part. Sci. **52**, 115 (2002) [arXiv:hep-ph/0202264].

[4] M. Moe and P. Vogel, Ann. Rev. Nucl. Part. Sci. **44**, 247 (1994).

[5] F. Piquemal, J. Phys. Conf. Ser. **120**, 052004 (2008).

[6] C. Giunti and M. Laveder, http://www.nu.to.infn.it/, Visited Apr. 11, 2010.