A New Custodial Symmetry to protect $Z_0 b_L \bar{b}_L$ coupling

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Abstract

We discuss $Z_0 b_L \bar{b}_L$ physics in this lecture.To set up a new model beyond the SM, after the Electroweak Precision Test(EWPT), and that the flavor changing currents appear only at charged sector at tree level(FCNC)[1], the remaining problem is there is a tension between keeping the $Zb_L\bar{b}_L$ couplings within the tolerable fluctuation around the well measured experimental constrain on bottom quark gauge currents and getting a sufficient larger heavy top quark mass. In other words, how to get a enough heavy top quark mass without giving larger correction to the $Zb\bar{b}$ vertex. Thus the $Z_0 b\bar{b}_L$ vertex has some dependence on the top quark physics, which is important due to "Top priority". One of the approaches to solving the problem is by introducing a new custodial symmetry, which is a combination of custodial symmetry and a Z_2 parity symmetry[2] to protects the $Zb_L\bar{b}_L$ couplings. This new custodial symmetry can also be imbedded into the adjoint representation of $SO(5)$ or fundamental representation of G_2 as in Pseudo-Goldstone models, in order to explore the dynamics of fundamental theory. A proof will be given, that the introducing of this new custodial symmetry is equivalent to make choices of one of two constraints on the gauge representation of fermions, by imposing one of which, the $Zb_L\bar{b}_L$ couplings will be protected after spontaneously symmetry breaking(SSB). As an illustrated example, I will give a concrete discussion based on left-right symmetric Higgsless model on warped AdS_5 gravitational background. A natural result of applying the constraint to the bulk gauge group of the model, is that a unconfined fundamental particle with fractional charge[3] appears simultaneously. This will be a distinguished prediction that can be tested at a near future Larger Hadron Collider(the LHC). Reference: arXiv:hep-ph/0607146

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I. ABSTRACT

II. GAUGE BOSONS ON CURVED SPACE-TIME

Consider a 5D gauge theory in a fixed gravitational background

$$ds^2 = (\frac{R}{z})^2(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2) \tag{1}$$

, where z is on the integral $[R, R^{'}]$. In the RS type model, the typical value of R is $\sim \frac{1}{M_{pl}}$, and $R^{'}$ is $\sim \frac{1}{TeV}$.

The action for a gauge theory on a fixed background is given by

$$S = \int d^5x\sqrt{g}[-\frac{1}{4}F^a_{MN}F^a_{PQ}g^{MP}g^{NG}] \tag{2}$$

,

By putting in the AdS_5 metric(Eq. 1), we get

$$S = \int dx^4 \int_R^{R^{'}} -\frac{1}{4}(\frac{R}{z})[F^a_{\mu\nu}{}^2 + 2F^a_{\mu 5}{}^2] \tag{3}$$

To quantize the gauge theory, we add the R_ξ gauge fixing term

$$S_{gf} = -\int d^4x \int_R^{R^{'}} dz \frac{1}{2\xi}\frac{R}{z}[\partial_\mu A^\mu - \xi\frac{z}{R}\partial_5(\frac{R}{z}A_5)]^2 \tag{4}$$

To read the propagator and the equation of motion of the gauge theory, we select the quadratic piece of gauge filed in the action,

$$\int d^4x \int_R^{R^{'}} dz \frac{R}{z}\frac{1}{2}A_\mu[(\partial^2 - \frac{z}{R}\partial_z(\frac{R}{z}\partial_z))\eta^{\mu\nu} - (1-\frac{1}{\xi})\partial_\mu\partial_\nu]A_\nu \tag{5}$$

By making mode expansion $A^a_\mu(x,z) \sim A^a_\mu\psi(z)e^{-ip\cdot x}$, the equation of motion for the gauge boson wave function $\psi(z)$ will be become:

$$[z\partial_z(\frac{1}{z}\partial_z) + p^2]\psi(z) = 0 \tag{6}$$

where $p^2 = M^2$,

the general solution for the above equation is of the form

$$\psi(z) = z[\alpha J_1(p_k z) + \beta Y_1(p_k z)] \tag{7}$$

where $J_1(z)$ and $Y_1(z)$ are the Bessel function of the first kind and the second kind separately.

III. FERMIONS ON CURVED SPACE-TIME

A general bulk fermion action in a curved space-time is

$$\mathcal{S}_f = \int dx^5 \sqrt{g}[\frac{i}{2}[\bar{\Psi}\Gamma^M D_M \Psi - \bar{D}_M \bar{\Psi}\Gamma^M \Psi] - m\bar{\Psi}\Psi] \tag{8}$$

where $\Gamma^M = e^M_a \gamma^a, \Gamma_M = e^a_M \gamma_a, \gamma^a = (\gamma^\mu, \gamma^5)$, a is for Minkowski metric, and $M = 0, 1, 2, 3, 5$ is used to label Curvered space-time metric $g_{MN} = e^a_M \eta_{ab} e^b_N, g^{MN} = e^M_a \eta^{ab} e^N_b$, where e^M_a is f*ü*nfbein, the generalization of the vierbein to higher dimensions. $D_M = \partial_M + \frac{1}{2}\omega^{ab}_M \sigma_{ab}$ is the covariant derivative including the spin connection term, which is

$$\omega^{ab}_M = \frac{1}{2} g^{RP}[e^a_R(\partial_M e^b_P - \partial_P e^b_M) - e^b_R(\partial_M e^b_P - \partial_P e^b_M)] + \frac{1}{4} g^{RP} g^{TS}(e^a_R e^b_T - e^b_R e^a_T)(\partial_S e^c_P - \partial_P e^c_S) e^d_M \eta_{cd} \tag{9}$$

and $\sigma_{ab} = -\frac{1}{4}[\gamma_a, \gamma_b]$ is the ordinary 2nd antisymmetric tensor.

For the AdS5 metric in conformal coordinate,

$$ds^2 = (\frac{R}{z})^2 [\eta_{\mu\nu} dx^\mu dx^\nu - dz^2]$$

from which, we find the f*ü*nfbein $e^a_M = \frac{R}{z}\delta^a_M, \quad e^M_a = \frac{R}{z}\delta^M_a, \sqrt{g} = (\frac{R}{z})^5$.

After some algebra reduction, we find

$$D_\mu \Psi = (\partial_\mu + \gamma_\mu \gamma_5 \frac{1}{2z})\Psi \qquad D_5 \Psi = \partial_5 \Psi \tag{10}$$

Thus the 5-D action is

$$\begin{aligned} \mathcal{S}_f = \int dx^4 dz (\tfrac{R}{z})^5 [\tfrac{i}{2}[\bar{\Psi}\Gamma^M \partial_M \Psi - \partial_\mu \bar{\Psi}\Gamma^M \Psi] \\ + \tfrac{i}{2}[\bar{\Psi}\Gamma^\mu \gamma_\mu \gamma_5 \tfrac{1}{2z}\Psi - (\gamma_\mu \gamma_5 \tfrac{1}{2z}\Psi)^\dagger \gamma^0 \Gamma^\mu \Psi] - m\bar{\Psi}\Psi] \end{aligned} \tag{11}$$

The fist term in the rectangular bracket is kinematic term while those in the second rectangular barcket is mass term.

Considering that $\Gamma^M = e^M_a \gamma^a = \frac{R}{z}\gamma^M$, we find the kinematic term as

$$\mathcal{S}_{fkin} = \int dx^4 dz (\frac{R}{z})^4 \frac{i}{2}\{[\bar{\Psi}\gamma^\mu \partial_\mu \Psi - \partial_\mu \bar{\Psi}\gamma^\mu \Psi] + [\bar{\Psi}\gamma^5 \partial_5 \Psi - \partial_5 \bar{\Psi}\gamma^5 \Psi]\}$$

the mass term is

$$\mathcal{S}_{fm} = \int dx^4 dz (\frac{R}{z})^5 [-m\bar{\Psi}\Psi] = \int dx^4 dz (\frac{R}{z})^4 [-\frac{c_f}{z}\bar{\Psi}\Psi]$$

where $m = \frac{c_f}{R}$ is the bulk Dirac mass in the unites of the AdS_5 curvature $\frac{1}{R}$.

we use the Chiral representation, namely

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \mu = 0, 1, 2, 3 \qquad \gamma^0 = \begin{pmatrix} 0 & -\mathbf{1}_2 \\ -\mathbf{1}_2 & 0 \end{pmatrix} \qquad \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix} \qquad \gamma^5 = \begin{pmatrix} i\mathbf{1}_2 & 0 \\ 0 & -i\mathbf{1}_2 \end{pmatrix} \tag{12}$$

the exact action is

$$\int dx^4 dz (\frac{R}{z})^4 [\frac{i}{2}\{[\bar{\Psi}\gamma^\mu \partial_\mu \Psi - \partial_\mu \bar{\Psi}\gamma^\mu \Psi] + [\bar{\Psi}\gamma^5 \partial_5 \Psi - \partial_5 \bar{\Psi}\gamma^5 \Psi]\} - \frac{c_f}{z}\bar{\Psi}\Psi]$$

In the Chiral representation, we introduce the Dirac fermions as

$$\Psi = \begin{pmatrix} \chi_\alpha \\ \bar{\psi}^{\dot{\alpha}} \end{pmatrix} \tag{13}$$

The dotted and undotted indices of a two component spinor are raised and lowered with the 2×2 antisymmetric tensors $\epsilon_{\alpha\beta} = i\sigma^2_{\alpha\beta}$ and $\epsilon_{\dot{\alpha}\dot{\beta}} = i\sigma^2_{\dot{\alpha}\dot{\beta}}$ and inverse $\epsilon^{\alpha\beta} = -i\sigma^2_{\alpha\beta}$ and $\epsilon^{\dot{\alpha}\dot{\beta}} = -i\sigma^2_{\dot{\alpha}\dot{\beta}}$:

$$\chi^\alpha = \epsilon^{\alpha\beta}\chi_\beta \quad \bar{\chi}_{\dot\alpha} = \epsilon_{\dot\alpha\dot\beta}\bar{\psi}^{\dot\beta} \tag{14}$$

Their product are the Lorentz invariant scalars,

$$\chi\psi = \chi^\alpha\psi_\alpha \quad \bar{\chi}\bar{\psi} = \bar{\chi}_{\dot\alpha}\bar{\psi}^{\dot\alpha} \tag{15}$$

which are symmetric:

$$\chi\psi = \psi\chi, \quad \bar{\chi}\bar{\psi} = \bar{\psi}c\bar{h}i \tag{16}$$

substitute into action, we find

$$S = \int d^5x(\tfrac{R}{z})^4\tfrac{i}{2}\{[-\bar{\chi}\bar{\sigma}^\mu\partial_\mu\chi - \psi\sigma^\mu\partial_\mu\bar{\psi} + \partial_\mu\psi\sigma^\mu\bar{\psi} + \partial_\mu\bar{\chi}\bar{\sigma}^\mu\chi] \tag{17}$$

$$+\tfrac{1}{2}[\psi\partial_z\chi + \partial_z\bar{\chi}\bar{\psi} - \bar{\chi}\partial_z\bar{\psi} - \partial_z\psi\chi] + \tfrac{c}{z}(\psi\chi + \bar{\chi}\bar{\psi})\} \tag{18}$$

where the coefficient $c_f = mR$, m is the bulk Dirac mass for the 4-component Dirac spinor.
The bulk EOM for the fermions in AdS_5 background are

$$-i\bar{\sigma}^\mu\partial_\mu\chi - \partial_z\bar{\psi} + \tfrac{c+2}{z}\bar{\psi} = 0 \tag{19}$$

$$-i\sigma^\mu\partial_\mu\bar{\psi} + \partial_z\chi + \tfrac{c-2}{z}\chi = 0 \tag{20}$$

Perform the KK mode decomposition as what we have done to the gauge filed,

$$\chi = \sum_k g_k(z)\chi_k(x), \quad \bar{\psi} = \sum_k f_k(z)\bar{\psi}_k(x) \tag{21}$$

where the 4D spinors χ_k and $\bar{\psi}_k$ satisfy the usual 4D Dirac equation with mass m_k:

$$i\bar{\sigma}^\mu\partial_\mu\chi_k - m_k\bar{\psi}_k = 0 \tag{22}$$

$$i\sigma^\mu\partial_\mu\bar{\psi}_k - m_k\chi_k = 0 \tag{23}$$

Then we find the coupled equations of bulk fermion wavefunctions f_k and g_k:

$$f_k^{'} + m_k g_k - \frac{c+2}{z}f_k = 0 \tag{24}$$

$$g_k^{'} - m_k f_k + \frac{c-2}{z}g_k = 0 \tag{25}$$

For the zero mode, the bulk fermion is decoupled and the corresponding wavefunctions are

$$f_0 = \alpha z^{2+c} \tag{26}$$

$$g_0 = \beta z^{2-c} \tag{27}$$

For the non-zero mode, the bulk fermion can be decoupled as the second order of differential equations:

$$f_k^{''} - \tfrac{4}{z}f_k^{'} + m_k^2 - \tfrac{c^2-c-6}{z^2}f_k = 0 \tag{28}$$

$$g_k^{''} - \tfrac{4}{z}g_k^{'} + m_k^2 - \tfrac{c^2+c-6}{z^2}g_k = 0 \tag{29}$$

$$\tag{30}$$

The solutions of the massive bulk fermions wavefunctions are

$$f_k(z) = z^{\frac{5}{2}}(A_k J_{c-\frac{1}{2}}(m_k z) + B_k Y_{c-\frac{1}{2}}(m_k z)) \tag{31}$$

$$g_k(z) = z^{\frac{5}{2}}(C_k J_{c+\frac{1}{2}}(m_k z) + D_k Y_{c+\frac{1}{2}}(m_k z)) \tag{32}$$

$$\tag{33}$$

The ordinary coupled first order of differential equations impose that $C_k = A_k, D_k = B_k$.

The left boundary terms in the variation of the action are:

$$\delta S_{\delta\bar{\chi}+\delta\psi} = \frac{1}{2}\int d^4x[(\frac{R}{z})^4(\delta\bar{\chi}\bar{\psi} - \delta\psi\chi)]|_R^{R'} \tag{34}$$

$$\delta S_{\delta\bar{\psi}+\delta\chi} = \frac{1}{2}\int d^4x[(\frac{R}{z})^4(-\bar{\chi}\delta\bar{\psi} + \psi\delta\chi)]|_R^{R'} \tag{35}$$

$$\tag{36}$$

The total boundary terms are

$$\delta S_{bound} = \frac{1}{2}\int d^4x(\frac{R}{z})^4[(\delta\bar{\chi}\bar{\psi} - \bar{\chi}\delta\bar{\psi}) + (\psi\delta\chi - \delta\psi\chi)]_R^{R'} \tag{37}$$

which agrees with the expression for flat space up the irrelevant factor of $\frac{R^4}{z^4}$, namely

$$\delta S_{bound-flat} = \frac{1}{2}\int d^4x[(\delta\bar{\chi}\bar{\psi} - \bar{\chi}\delta\bar{\psi}) + (\psi\delta\chi - \delta\psi\chi)]_0^L \tag{38}$$

The boundary conditions required are those can make the boundary variation of the action vanish. The simplest and most commonly adopted solutions are by fixing one of the two spinors (eg. ψ)to zero on the branes,

$$\psi|_{R,R'} = 0, \quad \delta\psi|_{R,R'}, \quad (\partial_z\chi + \frac{c-2}{z}\chi)|_{R,R'} = 0 \tag{39}$$

The last one is required by the bulk equations of the motion so that they are satisfied every where, including at the end points of the interval. In the chiral limit($c->2$), this is the usual *orbifold boundary*(If we assign a parity to χ and ψ under $y \to -y$, they have to have opposite parities in the bulk, due to the bulk term $\psi\partial_z\chi$, thus if ψ is chosen to be negative parity(Diriclet BC's,$\psi|_{R,R'} = 0$), then χ has to be positive (Neumann BC's,$\partial_z\chi|_{R,R'} = 0$)).

Because there are two constraints associated with the solutions of two first order differential equations, we need to impose one BC at each brane(the end of the interval or the fixpoint of the chain). The most general solution to vanish the boundary variation on the boundary is the two spinors χ and ψ are proportional to each other:

$$\psi|_{R,R'} = (M_\alpha^\beta\chi_\beta + N_{\alpha\dot{\beta}}\bar{\chi}^{\dot{\beta}})|_{R,R'} \tag{40}$$

where M, N are two matrices. Two simple solutions are

$$\psi|_{R,R'} = c(\chi_\alpha + i\sigma^\mu_{\alpha\dot{\beta}}\partial_\mu\bar{\chi}^{\dot{\beta}})|_{R,R'} \tag{41}$$

For fermions belonging to a *complex* representation of the gauge group, gauge invariance requires that $M = 0$. For those belonging to a *real* representation, M is allowed to be non-zero, and we can apply a mixed BC's for two spinors,

$$(a\psi + b\chi)|_{R,R'} = 0 \tag{42}$$

where a, b is normalized to unit.

IV. THE HIGGSLESS ELECTROWEAK SYMMETRY BREAKING MODEL

One of the most mystery of the particle physics is the mechanism for electroweak symmetry breaking(EWSB). The model here consists of bulk gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. On the UV(Planck) brane, $SU(2)_R \times U(1)_{B-L}$ is broken to $U(1)_Y$, while on the IR(TeV) brane, where EWSB happen, $SU(2)_L \times SU(2)_R$ is broken to $SU(2)_D$.

A. Gauge Sector

It has been pointed that extra dimensions may provide an alternative approach to unitarize the scattering of he massive gauge bosons via the exchange of a tower of massive KK gauge bosons[1]. It's convenient to choose[2] a group of boundary condition's (BC's) corresponding to the symmetry breaking pattern:

$$UV(z=R):\partial_z(A_\mu^{La}=0),\ A_\mu^{R1,2}=0,\ \partial_z(g_{5R}B_\mu+\tilde{g}_5A_\mu^{R3}=0),\ \tilde{g}_5B_\mu-g_{5R}A_\mu^{R3}=0$$
$$IR(z=R^{'}):\partial_z(A_\mu^{La}+A_\mu^{Ra})=0,\ A_\mu^{La}-A_\mu^{Ra}=0,\ \partial_zB_\mu=0 \tag{43}$$

The BC's for the A_5's are just the opposite to that of the corresponding combination of the 4D gauge fields.
As showed before, by make mode expansion, all of the gauge bosons' wave function will be of the form

$$\psi_k^A(z)=z(\alpha_k^AJ_1(qz)+\beta_k^AY_1(qz)) \tag{44}$$

we use k to label different KK modes, and A to the category of gauge bosons.
The KK expansion are[4]

$$B\mu(x,z)=\frac{1}{\tilde{g}_5}a_0\gamma_\mu(x)+\sum_{k=1}^{\infty}\psi_k^B(z)Z_\mu^k(x) \tag{45}$$
$$A_\mu^{L3}(x,z)=\frac{1}{g_{5L}}a_0\gamma_\mu+\sum_{k=1}^{\infty}\psi_k^{(L3)}(z)Z_\mu^k(x) \tag{46}$$
$$A_\mu^{R3}(x,z)=\frac{1}{g_{5R}}a_0\gamma_\mu+\sum_{k=1}^{\infty}\psi_k^{(R3)}(z)Z_\mu^k(x) \tag{47}$$
$$A_\mu^{L\pm}(x,z)=\sum_{k=1}^{\infty}\psi_k^{(L\pm)}(z)W_\mu^{k\pm}(x) \tag{48}$$
$$A_\mu^{R\pm}(x,z)=\sum_{k=1}^{\infty}\psi_k^{(R\pm)}(z)W_\mu^{k\pm}(x) \tag{49}$$
$$\tag{50}$$

where the 4D photon $\gamma(x)$ has a flat wavefunction due to the unbroken $U(1)_Q$ symmetry, and the massive charged and neutral gauge bosons are separately $W^{k\pm}(x)$ and $Z^k(x)$, the lowest of which are supposed to be the observed W and Z.
By substitute KK expansion into the BC's(81), we find the equations to determine the Kk tower for W and Z, separately,

$$(R_0-R_0^{'})(R_1-R_1^{'})=(R_1-R_0^{'})(R_1^{'}-R_0) \tag{51}$$
$$g_5^2(R_0-R_0^{'})(R_1-R_1^{'})=(g_5^2+2\tilde{g}_5^2)(R_1-R_0^{'})(R_1^{'}-R_0) \tag{52}$$

Note: to simplify the problem, we have assumed that $g_{5L}=g_{5R}$.

$$M_W^2\approx\frac{1}{R^{'2}\log\frac{R^{'}}{R}} \tag{53}$$
$$M_Z^2\approx\frac{g_5^2+2\tilde{g}_5^2}{g_5^2+\tilde{g}_5^2}\frac{1}{R^{'2}\log\frac{R^{'}}{R}} \tag{54}$$

To leading order in $\frac{1}{R}$ and for $\log\frac{R^{'}}{R}>>1$, the lightest mass spectrum for the charged and neutral gauge bosons are separately
By introducing the UV and IR brane kinetic terms

$$\mathcal{L}_{UVbrane}=-[\frac{r}{4}W_{\mu\nu}^{L2}+\frac{r^{'}}{4}\frac{1}{g_{5R}^2+\tilde{g}_5^2}(g_{5R}B_{\mu\nu}+\tilde{g}_5W_{\mu\nu}^{R3})^2]\delta(z-R) \tag{55}$$
$$\mathcal{L}_{IRbrane}=-\frac{R^{'}}{R}[\frac{\tau^{'}}{4}B_{\mu\nu}^2+\frac{\tau}{4}\frac{1}{g_{5R}^2+g_{5L}^2}(g_{5R}W_{\mu\nu}^L+g_{5L}W_{\mu\nu}^R)^2]\delta(z-R^{'}) \tag{56}$$
$$\tag{57}$$

and extend the BC's to

$$\text{For UV brane}(z=R):(\partial_z+rM^2)A_\mu^{La}=0,\ A_\mu^{R1,2}=0,\ (\partial_z+r^{'}M^2)(g_{5R}B_\mu+\tilde{g}_5A_\mu^{R3}=0),\ \tilde{g}_5B_\mu-g_{5R}A_\mu^{R3}=0 \tag{58}$$
$$\text{For IR brane}(z=R^{'}):(\partial_z-\tau M^2\frac{R^{'}}{R})(g_{5R}A_\mu^{La}+g_{5L}A_\mu^{Ra})=0,\ g_{5L}A_\mu^{La}-g_{5R}A_\mu^{Ra}=0,\ (\partial_z-\tau^{'}M^2\frac{R^{'}}{R})B_\mu=0 \tag{59}$$

where r and $r^{'}$ are kinetic term from the UV brane, while τ is the $L-R$ kinetic term and $\tau^{'}$ is the $B-L$ kinetic term on IR brane separately, all of above coefficients have dimensions of length.

To leading order in $\frac{1}{R}$ and for $\log\frac{R^{'}}{R}>>1$, the lightest mass spectrum for the charged and neutral gauge bosons are separately.

$$M_W^2 \approx \frac{2g_{5L}^2}{g_{5L}^2+g_{5R}^2}\frac{1}{1+\frac{r}{R\log\frac{R'}{R}}}\frac{1}{R'^2\log\frac{R'}{R}}(1+\frac{2g_{5L}^2}{g_{5L}^2+g_{5R}^2}\frac{1}{1+\frac{r}{R\log\frac{R'}{R}}}\frac{3}{8\log\frac{R'}{R}})(1-\frac{g_{5R}^2}{g_{5R}^2+g_{5L}^2}\frac{\tau}{r+R\log\frac{R'}{R}}) \tag{60}$$

$$M_Z^2 \approx \frac{2g_{5L}^2}{g_{5L}^2+g_{5R}^2}\frac{g^2+g'^2}{g^2}\frac{1}{1+\frac{r}{R\log\frac{R'}{R}}}\frac{1}{R'^2\log\frac{R'}{R}}(1+\frac{2g_{5L}^2}{g_{5L}^2+g_{5R}^2}\frac{g^2+g'^2}{g^2}\frac{1}{1+\frac{r}{R\log\frac{R'}{R}}}\frac{3}{8\log\frac{R'}{R}})[1-\frac{g_{5R}^2}{g_{5R}^2+g_{5L}^2}\frac{\tau}{r+R\log\frac{R'}{R}}(1-\frac{g_{5L}^2g'^2}{g_{5R}^2g^2})] \tag{61}$$

(62)

At the nonlinear level, the $B-L$ kinetic term τ' will contribute a negative contribution to the oblique parameters.

$$S \approx \frac{6\pi}{g^2\log\frac{R'}{R}} - \frac{8\pi}{g^2}(1-(\frac{g'}{g})^2)\frac{\tau'^2}{(R\log\frac{R'}{R})^2} \tag{63}$$

$$T \approx -\frac{2\pi}{g^2}(1-(\frac{g'}{g})^4)\frac{\tau'^2}{(R\log\frac{R'}{R})^2} \tag{64}$$

B. Fermion Sector

We are also hope to understand how fermion masses can be generated in the Higgsless model, in other words, the generation of fermion masses without a Higgs boson. The S parameter in the flat space version of the model has a large positive contribution and we would consider a fermions on the AdS_5 background[3].

The fermions have to be put into the bulk, since fermion need to connect with the TeV brane in order to feel the effect of EWSB. On the other hand, they can not simply put on the TeV brane, otherwise they will for multiplets of $SU(2)_D$.

The left handed SM fermions are assumed to form $SU(2)_L$ doublets and the right handed ones $SU(2)_R$ doublets(including right handed neutrino).

The smallest irreducible representation of the Lorentz group of a 5D bulk fermion is the Dirac spinor, which contains two 4D Weyl(two component) like spinors (like a 4D Dirac fermions in the view point of Dimensional Deconstruction). One have to make sure that in every 5D bulk fermion, there is only one single 4D Weyl spinor zero mode, which will be identified as the usual SM fermions. Because the usual gauge bosons couplings to light fermions is Left-Right unsymmetric, thus the zero modes for the light fermions have to be put in the TeV brane. While the theory on the TeV brane is Vector-like so the up and down type fermions will have degenerate masses, if naively add a mass term on the TeV brane. The up and down type fermions have to be splitting, and this can be achieved by mixing the right handed fermions with those localized on the Planck brane.

To discuss the symmetry breaking of fermions on the AdS_5 background, we have to apply the the boundary By imposing the conventional [3] Dirichlet BC's on both UV and IR branes:

$$\psi_R = 0 \quad \psi_{R'} = 0 \tag{65}$$

These BC's allow for a chiral zero mode in the χ sector while the ψ has to be vanishing, thus we have one zero mode wavefunction in the bulk27,

$$f_0 = 0, \quad g_0 = \beta z^{2-c} \tag{66}$$

where c is an arbitrary bulk mass coefficient, the choose of which will affect the location of zero mode, whether close to the UV brane(around $z = R$), or close to the IR(around $z = R'$). The normalization constant in front of the zero mode wavefunction can be determined by

$$\int_R^{R'} dz(\frac{R}{z})^5\frac{z}{R}\beta^2 z^{4-2c} = 1 \tag{67}$$

where $(\frac{R}{z})^5$ comes from volume $\sqrt{g}$ on AdS_5 background while $\frac{z}{R}$ from vierbein. We find the normalization constant

$$\beta = \frac{\sqrt{1-2c}}{R^c\sqrt{R'^{1-2c}-R^{1-2c}}} \tag{68}$$

The fermion zero mode is localized near the UV brane when $c > \frac{1}{2}$(Can be seen by sending IR brane to infinity $R' \to \infty$, $beta$ has to be converges) and it will be elementary, while localized near the IR one when $c < \frac{1}{2}$(by send

UV brane to infinity $R \to \infty$) and to be considered as composite bound states of the CFT modes. For $c = \frac{1}{2}$, the wavefunction is flat.

We can also impose the other Dirichlet BC's on both UV and IR branes:

$$\chi_R = 0 \quad \chi_{R'} = 0 \tag{69}$$

In which case, The BC's will allow for a chiral zero mode in the ψ sector while the χ has to be vanishing, thus we have one zero mode wavefunction in the bulk27,

$$f_0 = \alpha z^{2+c}, \quad g_0 = 0 \tag{70}$$

After normalization, we find the normalization constant of zero mode wave function of ψ as

$$\alpha = \frac{\sqrt{1+2c}}{R^{-c}\sqrt{R'^{1+2c} - R^{1+2c}}} \tag{71}$$

By the same trick, we find that a zero mode of ψ would have been localized on the UV brane for $c < -\frac{1}{2}$, and localized on the IR brane for $c > -\frac{1}{2}$. Note we can only have one zero mode for the chiral gauge theory. For $c = -\frac{1}{2}$, the wavefunction is flat.

For the Left-Right handed Higgsless model at hand, we have two $SU(2)$ doublet Dirac fermions for the leptons and two separately for the quarks in the bulk for each generation. Each Dirac fermion has a bulk mass $c_{L,R}$ and a Dirac mass M_D on the TeV brane, which mixes the left and right handed bulk fermions. In addition, we assume that there is a Dirac fermion localized on the UV brane that mixes with the ψ_R, in order to be able to split the masses of the up and down type fermions.

In a left-right handed symmetric model, the left and right handed fermions are in the representation of $(2, 1, \frac{1}{2}(B-L))$ and $(1, 2, \frac{1}{2}(B-L))$ of the bulk gauge group $SU(2)_L \times SU(2)_R \times U(1)_{\frac{B-L}{2}}$ respectively. Because the bulk fermion are Dirac fermions and thus every chiral SM fermions is doubled(adding right handed neutrino too).

1. *Lepton sector*

The left and right handed doublet of lepton are in representation of $(2, 1, -\frac{1}{2})$ and $(1, 2, -\frac{1}{2})$ of the bulk gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ respectively.

$$\begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \quad \begin{pmatrix} \nu_R \\ e_R \end{pmatrix} \tag{72}$$

where

$$\nu_L = \begin{pmatrix} \chi_{\nu_L} \\ \bar{\psi}_{\nu_L} \end{pmatrix} \quad e_L = \begin{pmatrix} \chi_{e_L} \\ \bar{\psi}_{e_L} \end{pmatrix}, \qquad \nu_R = \begin{pmatrix} \chi_{\nu_R} \\ \bar{\psi}_{\nu_R} \end{pmatrix} \quad e_R = \begin{pmatrix} \chi_{e_R} \\ \bar{\psi}_{e_R} \end{pmatrix} \tag{73}$$

$$L = \begin{pmatrix} \chi_{\nu_L} \\ \chi_{e_L} \end{pmatrix} \quad R = \begin{pmatrix} \chi_{\nu_R} \\ \chi_{e_R} \end{pmatrix} \tag{74}$$

eventually L correspond to the SM $SU(2)_L$ doublet, while R would correspond to the "SM(extended)" right handed doublet(i.e., including right handed electron and the "extra" right handed neutrino).

and

$$\bar{L} = \begin{pmatrix} \psi_{\nu_L} \\ \psi_{e_L} \end{pmatrix} \quad \bar{R} = \begin{pmatrix} \psi_{\nu_R} \\ \psi_{e_R} \end{pmatrix} \tag{75}$$

$\bar{L}$ is $SU(2)_L$ antidoublet partner and $\bar{R}$ is that of R, needed to for a complete Left and Right handed 5D Dirac spinor separately.

2. *Quark sector*

The left and right handed doublet of quark are in representation of $(2,1,\frac{1}{6})$ and $(1,2,\frac{1}{6})$ of the bulk gauge group $SU(2)_L \times SU(2)_R \times U(1)_{\frac{B-L}{2}}$ respectively.

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad \begin{pmatrix} u_R \\ d_R \end{pmatrix} \tag{76}$$

where

$$u_L = \begin{pmatrix} \chi_{u_L} \\ \bar{\psi}_{u_L} \end{pmatrix} \quad d_L = \begin{pmatrix} \chi_{d_L} \\ \bar{\psi}_{d_L} \end{pmatrix}, \qquad u_R = \begin{pmatrix} \chi_{u_R} \\ \bar{\psi}_{u_R} \end{pmatrix} \quad d_R = \begin{pmatrix} \chi_{d_R} \\ \bar{\psi}_{d_R} \end{pmatrix} \tag{77}$$

$$Q_L = \begin{pmatrix} \chi_{u_L} \\ \chi_{d_L} \end{pmatrix} \quad (U_R, D_R) = (\psi_{u_R}, \psi_{d_R}) \tag{78}$$

eventually Q_L correspond to the SM $SU(2)_L$ quark doublet, while ψ_{u_R} and ψ_{d_R} would correspond to the SM right handed quark singlet.
and

$$\bar{Q}_L = \begin{pmatrix} \psi_{u_L} \\ \psi_{d_L} \end{pmatrix} \quad (\bar{U}_R, \bar{D}_R) = (\chi_{u_R}, \chi_{d_R}) \tag{79}$$

are corresponding anti-let partners needed to form the completes 5D spinors.
Finally a localized 4D Dirac spinor that coupled to ψ_{u_R} on the UV brane $SU(2)_L \times U(1)_Y$ at R, has to be included,

$$(\xi_{u_R}, \bar{\eta}_{u_R}) \tag{80}$$

3. *Applying Boundary Conditions for Fermions*

It's convenient to choose[3] a group of boundary condition's (BC's) corresponding to the symmetry breaking pattern:

$$\begin{aligned} UV(z=R): \psi_L = 0,\ \chi_R - i\kappa\sigma^\mu\partial_\mu\bar{\psi}_R = 0, \\ IR(z=R^{'}): \bar{\psi}_L + M_D R^{'}\bar{\psi}_R = 0,\ \chi_R - M_D R^{'}\chi_L = 0 \end{aligned} \tag{81}$$

C. Gauge Fermion interaction term

The bulk gauge interaction in the unitary gauge comes from

$$S_{kin} = \int d^5x (\tfrac{R}{z})^4 \tfrac{i}{2}\{[-\bar{\chi}\bar{\sigma}^\mu D_\mu\chi - \psi\sigma^\mu D_\mu\bar{\psi} + D_\mu\psi\sigma^\mu\bar{\psi} + D_\mu\bar{\chi}\bar{\sigma}^\mu\chi] \tag{82}$$

$$+\tfrac{1}{2}[\psi D_z\chi + D_z\bar{\chi}\bar{\psi} - \bar{\chi}D_z\bar{\psi} - D_z\psi\chi]\} \tag{83}$$

where

$$\begin{aligned} D_\mu = \partial_\mu - ig_{5L}A_\mu^{La}T_L^a - ig_{5R}A_\mu^{Ra}T_R^a - i\tilde{g}_5\tfrac{Y}{2}B_\mu \\ D_z = \partial_z \end{aligned} \tag{84}$$

Note that the 5th covariant derivative is just the ordinary derivative in the unitary gauge, while for a general gauge choice such as R_ξ gauge, it will be

$$D_z = \partial_z - ig_{5L}A_5^{La}T_L^a - ig_{5R}A_5^{Ra}T_R^a - i\tilde{g}_5\frac{Y}{2}B_5 \tag{85}$$

where A_5^L, A_5^R and B_5 are separately the 5th components of the bulk gauge field and play roles of the goldstone bosons relative to the corresponding gauge bosons in 4D viewpoint.

The UV brane kinetic terms come from

$$S_{UVkin} = \int d^5x(\tfrac{R}{z})^4(-i)\kappa\delta(z-R)\psi\sigma^\mu D_\mu\psi \tag{86}$$

or

$$S_{UVkin} = \int d^5x(\tfrac{R}{z})^4(-i)\kappa\delta(z-R)\bar{\chi}\bar{\sigma}^\mu D_\mu\chi \tag{87}$$

but can not both.

The κ will depend on the flavor in each generation, with κ_u,κ_d labeled separately. Because $SU(2)_L$ symmetry is unbroken on UV brane, we have

$$\kappa_{u_L} = \kappa_{d_L} \tag{88}$$

The left κ_{u_R} and κ_{d_R} are independent.

D. $Zb\bar{b}$ vertex

The remaining problem in this kind of Higgsless model is how to get a large enough top quark mass without messing up the $Zb_l\bar{b}_l$ couplings. One of the approaches is suggested by a combination of custodial symmetry and a $L \longleftrightarrow R$ parity symmetry that protects the $Zb_l\bar{Z}_l$ couplings. This enhanced custodial symmetry suppresses corrections to the $Zb_l\bar{b}_l$ vertex. A scheme[7] proposed to solve the problem is as follows: The left-handed(LH) top and bottom quarks are part of a bi-doublet of $SU(2)_L \times SU(2)_R$, while the right-handed(RH) top is a singlet and the RH bottom is part of an $SU(2)_R$ triplet.

In the SM, the Higgs sector has an $SU(2)_L \times SU(2)_R$ symmetry which is broken down to a diagonal $SU(2)_D$ custodial symmetry by the Higgs Vev. $SU(2)_L$ is gauge symmetry, while $SU(2)_R$ is a global symmetry, which is broken by Yukawa couplings and the hypercharge gauge coupling. Yukawa couplings would not break the custodial symmetry if the RH fermions were doublets of $SU(2)_R$, but this is impossible, otherwise the top and bottom quark would have to be mass degenerate. In the Higgsless model at hand, the SM is embedded in a 5D Anti-de Sitter space (AdS_5), a custodial symmetry can be achieved by incorporating a bulk gauge symmetry $SU(2)_L\times SU(2)_R\times U(1)_{\frac{B-L}{2}}$. Here the $SU(2)_R$ symmetry is gauge symmetry. On the UV brane, the $SU(2)_R \times U(1)_{B-L}$ is broken to $U(1)_Y$, which is in according with the AdS/CFT correspondence, namely a global symmetry of the strongly coupled CFT corresponds to a gauge symmetry in AdS_R. On the TeV Brane, $SU(2)_L \times SU(2)_R$ is broken down to an $SU(2)_D$ custodial symmetry.

For the 3rd generation quarks,

$$\psi_L = \begin{pmatrix} t_L \\ b_L \end{pmatrix} \quad \psi_R = \begin{pmatrix} t_R \\ b_R \end{pmatrix} \tag{89}$$

where

$$t_L = \begin{pmatrix} \chi_{t_L} \\ \bar{\psi}_{t_L} \end{pmatrix} \quad b_L = \begin{pmatrix} \chi_{b_L} \\ \bar{\psi}_{b_L} \end{pmatrix}, \qquad t_R = \begin{pmatrix} \chi_{t_R} \\ \bar{\psi}_{t_R} \end{pmatrix} \quad b_R = \begin{pmatrix} \chi_{b_R} \\ \bar{\psi}_{b_R} \end{pmatrix} \tag{90}$$

The SM fermions can be reproduced by the assignment the following BC's(UV, IR):

$$\chi_L = \begin{pmatrix} \chi_{t_L}(+,+) \\ \chi_{b_L}(+,+) \end{pmatrix} \quad \bar{\psi}_L = \begin{pmatrix} \bar{\psi}_{t_L}(-,-) \\ \bar{\psi}_{b_L}(-,-) \end{pmatrix}, \qquad \chi_R = \begin{pmatrix} \chi_{t_R}(-,-) \\ \chi_{b_R}(\mp,-) \end{pmatrix} \quad \bar{\psi}_R = \begin{pmatrix} \bar{\psi}_{t_R}(+,+) \\ \bar{\psi}_{b_R}(\pm,+) \end{pmatrix}, \tag{91}$$

where $+$ stands for a Neumann BC and $-$ stands for a Dirichlet BC. By imposing BC's on the branes, the SM zero modes are reproduced. The LH and RH components of a 5D bulk fermion must always have opposite BC's.

There is a tension between having heavy top quark and small corrections to the couplings of the LH b with the Z boson. In order to enhance the mass of top, one has to localize the it as close as possible to the IR brane.However EWSB also happens there which distorts the wavefunctions of W and Z. On the other hand, a massive LH b' quark presents in the $SU(2)_R$ doublet that also contains the RH t, which mixes with the b via the TeV Dirac mass responsible for the t mass. Because a heavy top mass will require Dirac mass on the TeV brane to be very large, a larger mixing between b and t is generated.

The combination of custodial symmetry and $L \longleftrightarrow R$ parity symmetry forms a new custodial symmetry $SU(2)_D \times P_{LR}$, to protect the $Zb_l\bar{b}_l$ couplings. Note P_{LR}, which is the discrete parity interchanging the two $SU(2)$'s, is already a new symmetry beyond the SM. We can assume a beyond the SM sector, with a global $O(4) \sim SU(2)_L \times SU(2)_R \times P_{LR}$ symmetry, then is broken to $O(3) \sim SU(2)_D \times P_{LR}$. The breaking $SU(2)_R \times U(1)_{\frac{1}{2}(B-L)} \to U(1)_Y$ on the UV brane breaks P_{LR}, so that this new symmetry is only approximate(Left-Right non-symmetric).

If fermion is a eigenstate of P_{LR} with eigenvalue $+1$, then

$$T_L = T_R, \quad T_R^3 = T_R^3 \tag{92}$$

When global $O(4)$ is broken to $O(3)$, P_{LR} is still kept, however $SU(2)_L \times SU(2)_R$ is broken to diagonal custodial symmetry $SU(2)_D$, which protect the charge $Q_{L+R} = Q_L + Q_R$. On the other hand, the shifts in Q_L and Q_R must be equal since parity invariance.

$$\delta Q_L + \delta Q_R = 0, \quad \delta Q_L = \delta Q_R \tag{93}$$

thus the charges are individually protected,

$$\delta Q_L = \delta Q_R = 0 \tag{94}$$

While $Q = T^3 + Y$, this means the W_L^3 coupling is unchanged $\delta T^3_{L,R} = \delta R^3_R$. Take a special case as an example, fermions with $T_L^3 = T_R^3 = 0$ will never be able to couple to W_L^3 at all. More generally, $T_L^3 = T_R^3$

Remember that b_L belong to representation of $(2, 1, \frac{1}{6})$, in the bulk gauge group $SU(2)_L \times SU(2)_R \times U(1)_{\frac{1}{2}(B-L)}$.

$$T_L = \frac{1}{2}, T_R = 0; T_L^3 = -\frac{1}{2}, T_R^3 = 0 \tag{95}$$

satisfies neither of the requirements by new custodial symmetry as follows:

$$(a) T_L = T_R, \delta Q_L = \delta (Q)_R = 0 \quad or \quad (b) T_L = T_R = 0, \delta Q_L = \delta Q_R = 0 \tag{96}$$

By including the new P_{LR} parity symmetry, we can use an assignment for b_L quantum numbers:

$$T_L = T_R = \frac{1}{2}, T_L^3 = T_R^3 = -\frac{1}{2} \tag{97}$$

This will guarantee that $Zb_L\bar{b}_L$ doesn't receive correction from the new physics beyond SM. This also simultaneously assign top the quantum numbers in order to put t_L in a $SU(2)_L$ doublet with b_L:

$$T_L = T_R = \frac{1}{2}, T_L^3 = -T_R^3 = \frac{1}{2} \tag{98}$$

However, we know in SM, the corresponding quantum number for top is

$$T_L = \frac{1}{2}, T_R = 0, T_L^3 = \frac{1}{2}, T_R^3 = 0 \tag{99}$$

Thus we have to give up $SU(2)_L$ doublet representation. We can extend b_L in the representation $(4, \frac{2}{3})$ of global symmetry $O(4) \times U(1)_{\frac{1}{2}(B-L)}$ Namely, this can be done by embedding b_L, t_L into a bi-doublet $(2, 2, \frac{2}{3})$ of bulk gauge symmetry $SU(2)_L \times SU(2)_R \times U(1)_{\frac{1}{2}(B-L)}$. Consequently, the RH fermions can be either singlets or triplets of $SU(2)_L$ and/or $SU(2)_R$.

One of two possible selection of the quantum numbers of 3rd fermion representation in $SU(2)_L \times SU(2)_R \times U(1)_{\frac{1}{2}(B-L)}$, can be

$$(t_L, b_L) \in (2, 2, \frac{2}{3}) \sim \Psi_L, \quad b_R \in (1, 3, \frac{2}{3}) \sim \Psi_R, \quad t_R \in (1, 1, \frac{2}{3}) \sim t_R \tag{100}$$

$$\Psi_L = (q_L, Q_L) = \begin{pmatrix} t_L & X_L \\ b_L & T_L \end{pmatrix}, \quad \Psi_R = \begin{pmatrix} X_R \\ T_R \\ b_R \end{pmatrix}, \quad t_R \tag{101}$$

where all the fermion fields are bulk fields.

TABLE I: Quantum numbers of the bulk fermions

Fields	X_L	T_L	t_L	b_L	X_R	T_R	b_R	t_R
T_L^3	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0
T_R^3	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	1	0	-1	0

The right handed top t_R is in a bi-singlet, which doesn't contain any field that can mix with b_L: in other words, the t mass will not induce any mixing in the b sector.

Note: Because

$$Q = T_L^3 + Y, \quad Y = T_R^3 + \frac{1}{2}(B - L) \tag{102}$$

It's easy to check that the extra fermions T and X will carry a fractional charge $\frac{2}{3}$ and $\frac{5}{3}$, separately.

The quantum numbers of various fermions fields are shown in talbe

In order to be able to enhance the new custodial symmetry to protect the $Zb_L\bar{b}_L$, $\Psi_R \sim (1, 3, \frac{2}{3})$ should be completed to a full $O(4) \sim SU(2)_L \times SU(2)_R \times P_{LR}$ representation $\sim (3, 1, \frac{2}{3}) \oplus (1, 3, \frac{2}{3})$. In other words, we need to complete the representation with a $SU(2)_L$ triplet, it component with $T_L = Q_R - Y = -1$ will also mix with b_L, which will cancel out the contribution of the $SU(2)_R$ triplet(whose component with $T_R^3 = Q_R - Y = -1$) to $Zb_L\bar{b}_L$. This can be introduced by mixing t_L and T_L in $(2, 2, \frac{2}{3})$, to combine as a $SU(2)_L$ triplet.

The BC's those we impose are (UV,IR):

$$\chi_{q_L}(+,+), \quad \chi_{Q_L}(-,+), \quad \psi_{X_R}(-,+), \quad \psi_{T_R}(-,+), \quad \psi_{b_R}(+,+), \quad \psi_{r_R}(+,+) \tag{103}$$

where $+$ and $-$ are separately represents Neumann and Dirichlet BC.

The localized mass term on the IR brane is of the form:

$$\mathcal{L}_{IR} = M_3[\frac{1}{\sqrt{2}}T_R(t_L + T_L) + b_R b_L + X_R X_L] + \frac{M_1}{\sqrt{2}} t_R(t_L - T_L) + H.c. \tag{104}$$

The bulk gauge symmetry $SU(2)_L \times SU(2)_R$ is broken to the unbroken $SU(2)_D$ on the IR brane, t_L and T_L will be mixed, and we get a $SU(2)_L$ singlet $t_L^{'}$ as well as a $SU(2)_L$ triplet $\Psi_L^{'}$:

$$\frac{t_L - T_L}{\sqrt{2}} = t_L^{'} \in (1, 1, \frac{2}{3}), \quad \begin{pmatrix} X_L \\ \frac{1}{\sqrt{2}}(t_L + T_L) \\ b_L \end{pmatrix} = T_L^{'} \in (3, 1, \frac{2}{3}) \tag{105}$$

The $SU(2)_L$ triplet $\Psi_L^{'}$ is just what we need complete with Ψ_R to form the global $O(4)$.

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[1] See DC's review on $D_0\bar{D}_0$ mixing for more detail

[2] Note: this Z_2 symmetry is very similar to the $U(1)_A$ symmetry in QCD chiral gauge theory, the eigenfunction is just γ^5 with $\pm$ as the eigenvalue of Right and Left parity.

[3] Note: particle with fractional charge in 2+1 spacetime will have fractional statistics due to topological quantization, which can be described by Jones Ploynomials for $SU(N)$ symmetry. While, in the real world 3+1 spacetime, there is no annoy due to the angular momentum continuation. What we confront firstly in particle physics is the $U(1)_A$ problem in QCD. The $U(1)_A$ symmetry is spontaneously broken(SSB) at tree level but explicit broken at loop level due to chiral anomaly from fermions' triangle loops.In massless quark limit, by introducing a gauge-variant but conserved 2+1 Chern-Simons term, a conserved axial currents is reached, thus the $U(1)_A$ is SSB again and some goldstone boson must appear due to goldstone theorem. However this is no observation for this goldstone when analyzing QCD hadron spectrum. Luckily Kogut and Susskind found that the missing (pseudo-)goldstone(one of the so called goldstone dipole, with only derivative term at tree level) degree of freedom due to SSB coupled only to the gauge-variant term, which is not physically observable. Another brilliant solution to the $U(1)_A$ problem is solved by t'Hooft by introducing instanton. See Yuhsin's note

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