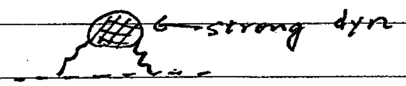
GGM

0801.3278	Patrick's
0812.3668	+ 3rd TASI
0901.1326	lecture

— Motivation :

- Want to know the param-space of GM.
- Want to calculate the GM ~~SUSY~~ when having strong dynamics in it.

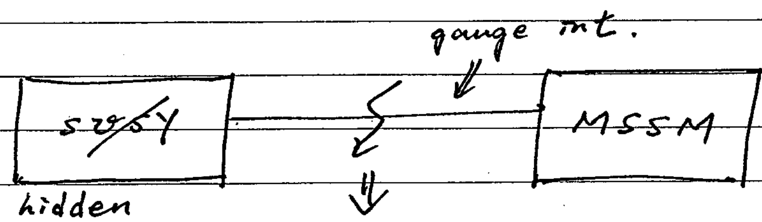
e.g. 

$\sim m^2_{soft}$

— Concepts :

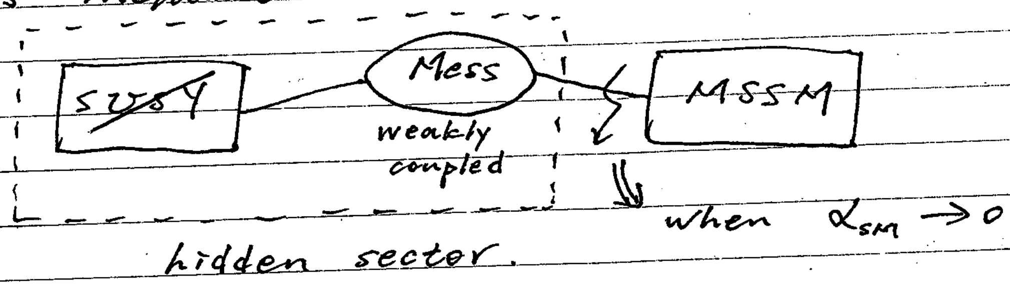
1° Giving ~~a~~ the most general def of GM.
2° Include all the hidden sector info into the consv current of the global symm.
3° Cal the possible correlation functions of the curren
4° couple the currents to the VSF (gauge the global symm
5° Doing the perturbative cal in g_{SM}, includes all the hidden sector info into global current correlation fun
6° Derive general rules of the soft terms, which indicates the param-space of GGM.
7° Give an existence proof of the model that covers ~~all~~ the whole param-space.

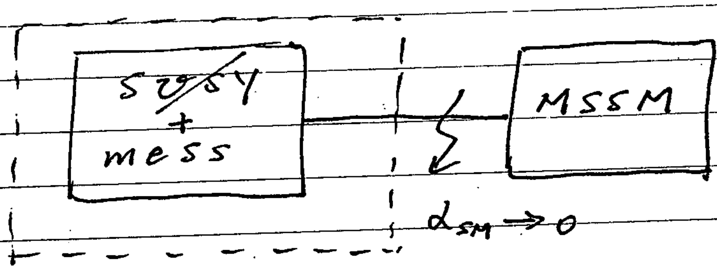
— The general def of GM.



decouple when $\alpha_{MSSM} \to 0$

This includes:

1° 

2° 

Direct GM

3° Case 2° with strongly coupled DSB.

4° Semi-Direct GM (mess charged under strong dyn., but doesn't join $\cancel{SUSY}$.)

⋮

The $\cancel{SUSY}$ sector:

Two assumptions:

1° $\exists$ one characteristic scale M, s.t.

$$\begin{cases} \cancel{SUSY} \Rightarrow \text{distance} > 1/M \\ SUSY \text{ restored} \Rightarrow \quad " \quad < 1/M \end{cases}$$

2° The global symm is large enough s.t. can gauge inside it to get G_{SM}.

(the global symm gives constraints on m_{soft})

The Current of the global symm:

1° Global symm transf:

$$S \to S + \partial_\mu J^\mu \quad , \quad \partial_\mu J^\mu = 0 \text{ if symm}$$

also, $\delta\phi_i = i\varepsilon T_{ij}\phi_j$, $\delta\phi_i^* = -i\varepsilon T_{ij}\phi_j^*$

2° like what we do for SUSY gauge transf, enlarge the global transf to XSF:

$$\Rightarrow \delta\phi_i = i\varepsilon \underbrace{\Lambda}_{XSF} T_{ij}\phi_j \ , \quad \delta\phi_i^* = -i\varepsilon \underbrace{\Lambda^*}_{\text{anti-}XSF} T_{ij}\phi_j^*$$

(want to do this since we'll gauge the global symm for MSSM in the end.)

3° Super-potential is inV under this transf. (XSF)

4° Kähler term is NOT inV!

$$S_{\text{Kähler}} \longrightarrow S_{\text{inv}} + \underbrace{\int d^4\theta\, (a\Lambda + b\Lambda^\dagger)\, \mathcal{J}}_{\neq 0 \text{ in general}}$$

some SF.

5° parametrize $\Lambda = \bar{D}^2 g$, $\Lambda^\dagger = D^2 f$.

where g & f are general SF. This gives

$$\bar{D}\Lambda = \bar{D}\bar{D}\bar{D}\, g = 0 \quad , \quad D\Lambda^\dagger = DDD f = 0.$$

satisfy ~~to~~ the XSF constraint.

6° $\delta S = \int d^4\theta\, (a\bar{D}^2 g + b D^2 f)\, \mathcal{J}$

$$\begin{pmatrix}\text{intg by} \\ \text{parts}\end{pmatrix} = \int d^4\theta\, (a g \bar{D}^2 \mathcal{J} + b f D^2 \mathcal{J})$$

7° If S is inV under the global symm, $\delta S = 0$ for generic f & g. This ensures

$$\boxed{D^2 \mathcal{J} = \bar{D}^2 \mathcal{J} = 0} \quad (1)$$

Definition of Real Linear SF.

expand into super-space:

$$\boxed{\begin{aligned}\mathcal{J} = & \ J + i\theta j - i\bar{\theta}\bar{j} - \theta\sigma^\mu\bar{\theta}\, j_\mu \\ & + \tfrac{1}{2}\theta\theta\bar{\theta}\,\bar{\sigma}^\mu\partial_\mu j - \tfrac{1}{2}\bar{\theta}\bar{\theta}\theta\,\sigma^\mu\partial_\mu\bar{j} - \tfrac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\,\Box J\end{aligned}} \quad (2)$$

also, $\boxed{\partial^\mu j_\mu = 0}$ (3)

8° Since gauging the global symm gives Gauge symm in MSSM,

$$\boxed{\text{$\mathcal{J}$ is inside the gauge current in MSSM.}}$$

Example: $\mathcal{L}_{k\ddot{a}h} = \int d^4\theta \left(\Phi^\dagger e^{2qV} \Phi + \tilde{\Phi}^\dagger e^{-2qV} \tilde{\Phi} \right)$

(U(1) toy model)

Under $U(1)$ global transf :

$$\delta \Phi = i\varepsilon (2g) \Lambda \Phi \qquad \delta \Phi^* = -i\varepsilon (2g) \Lambda^+ \Phi^*$$

$$\delta \tilde{\Phi} = i\varepsilon (-2g) \Lambda \tilde{\Phi} \qquad \delta \tilde{\Phi}^* = -i\varepsilon (-2g) \Lambda^+ \tilde{\Phi}^*$$

$$\delta \mathcal{L}_{k\ddot{a}h} = \int d^4\theta \; 2gi\varepsilon (\Lambda - \Lambda^+) \underbrace{(\Phi^*\Phi - \tilde{\Phi}^*\tilde{\Phi})}_{\Downarrow}$$

~~Define~~ Want the component in $\Downarrow$ s.t.

$$D^2 \, m = \bar{D}^2 \, m = 0$$

Expand $(\Phi^*\Phi - \tilde{\Phi}^*\tilde{\Phi})$ into ϕ & ψ's & match the component in (2).

$$\Rightarrow \begin{cases} J(x) = \phi^*\phi(x) - \tilde{\phi}^*\tilde{\phi}(x) \\ j(x) = -\sqrt{2}\, i \, (\phi^*\psi - \tilde{\phi}^*\tilde{\psi}) \\ \bar{j}(x) = \sqrt{2}\, i \, (\phi\bar{\psi} - \tilde{\phi}\bar{\tilde{\psi}}) \\ j_\mu(x) = i(\phi\partial_\mu\phi^* - \phi^*\partial_\mu\phi - \tilde{\phi}\partial_\mu\tilde{\phi}^* + \tilde{\phi}^*\partial_\mu\tilde{\phi}) \\ \qquad + \psi\sigma_\mu\bar{\psi} - \tilde{\psi}\sigma_\mu\bar{\tilde{\psi}} \end{cases}$$

Mess-parity:
$\phi \to v^* \tilde{\phi}^*$
$\tilde{\phi} \to \tilde{v}\, \phi^*$. then $J \to -J$

(can use the expansion of the Kähler term $\Phi^+\Phi$ in the textbooks to get the result).

— Correlation func. of the J-components :

(Assume $U(1)$ here)

1° 1pt func : (FI term-like)

$$\boxed{\langle J \rangle = \xi}\quad , \ \xi \neq 0 \text{ for abelian.}$$

2° 2pt func :

only 4 of them under $\begin{cases} \text{Lorentz inv} \\ \partial_\mu j^\mu = 0 \end{cases}$

$$\boxed{\begin{aligned}
\langle J(x)\, J(0) \rangle &= \frac{1}{x^4}\, C_0(xM) \\
\langle j_\alpha(x)\, \bar{j}_{\dot\alpha}(0) \rangle &= -i\, \bar\sigma^\mu_{\alpha\dot\alpha}\, \partial_\mu \left(\frac{1}{x^4}\, C_{1/2}(xM) \right) \\
\langle j_\mu(x)\, j_\nu(0) \rangle &= \left(\eta_{\mu\nu}\, \partial^2 - \partial_\mu \partial_\nu \right) \left(\frac{1}{x^4}\, C_1(xM) \right) \\
\langle j_\alpha(x)\, j_\beta(0) \rangle &= \varepsilon_{\alpha\beta}\, \frac{1}{x^5}\, B_{1/2}(x^2M^2)
\end{aligned}}$$

We don't know the exact form of C's & B in general. The thing we know is :

- C's $\in \mathbb{R}e$, $B \in \mathbb{C}$
- $\boxed{\begin{array}{l} \text{when } x \to 0 . \\ \text{a) } C_0 = C_{1/2} = C_1 = C \\ \text{b) } B_{1/2} = 0 \end{array}}$

Reason for a) :

when doing OPE with the operators ~~is~~ in the form $\mathcal{O}^+\mathcal{O}$, it's always.

$$\mathcal{O}^+\mathcal{O}(x) \sim (\underset{\sim}{\mathbb{1}} + 0x + 0x^2 + \cdots) \times \boxed{\text{decided by the dim in UV}}$$

$\therefore$ The UV result is totally fixed by the dimensional analysis, the C's are all the same constant.

Reason for b):

$\langle j_\alpha j_\beta \rangle$ is not in the type $\mathcal{O}^+\mathcal{O}$, also, when $x \to 0$, SUSY restored, $m_{gaugino} = 0$.

& $\langle j_\alpha j_\beta \rangle \sim m_{gaugino} \sim 0$ (we'll see this later).

~~Remarks~~

Remarks:

1° Can also write the correlation funcs into momentum space. See eq.(2.12) in GGM-paper

The UV limit gives $\tilde{C}_s \sim c \ln \frac{\Lambda}{M}$ (momentum space), $\tilde{B} = 0$, which depds on the generic scale M & the regular-scale Λ.

2° Can also write everything into super charges θ_α. One can use this expression to

show the finiteness of the correlation funcs.
see. sec 2.1 in 0812.3668.

Gauge the Symm. (global)

1° Gauge the symm, $\partial_\mu \to D_\mu$, gives the coupling.

2° $\mathcal{L}_{int} = \underset{\sim}{2 g_{SM}} \int d^4\theta \, \mathcal{J} \underset{=}{\mathcal{V}}_{VSF}$

Using WZ gauge:

$$\mathcal{V} = -\theta\sigma^\mu\bar{\theta} v_\mu + i\theta\theta\bar{\theta}\bar{\lambda} - i\bar{\theta}\bar{\theta}\theta\lambda + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta} D(x)$$

$$\int d^4\theta \, \mathcal{J}\mathcal{V} = \frac{1}{2} J D - i\lambda j + i\bar{\lambda}\bar{j} + j_\mu v^\mu$$

3° Doing Path-intgl

$$\int d\phi_{visible} \int d\phi_{hidden} \; e^{S_{free}} e^{S_{int}}$$

$$e^{S_{int}} = 1 + g_{SM} \int d^4\theta \, \mathcal{J}\mathcal{V} + \frac{1}{2} g_{SM}^2 \left(\int d^4\theta \, \mathcal{J}\mathcal{V}\right)^2 + \cdots$$

Can do perturbation in g_{SM}.

$$g_{SM}^2 \left(J D + j \lambda + \bar{j} \bar{\lambda} + j_\mu v^\mu \right)^2$$

$$\Rightarrow \delta \mathcal{L}_{eff}^{g^2} = \frac{1}{2} g^2 \tilde{C}_0 D^2 + g^2 \tilde{C}_{1/2}\, i \lambda \not\partial \bar{\lambda} + \frac{1}{4} g^2 \tilde{C}_1 F_{\mu\nu} F^{\mu\nu}$$

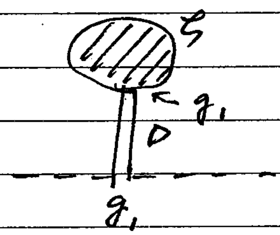
$$+ \underbrace{\frac{1}{2} g^2 \left(M \tilde{B}_{1/2} \lambda \lambda + c.c. \right)}_{\text{Gaugino mass term!}}$$

The Soft ~~term~~ masses :

1° Tree-level $m_{gaugino}$:

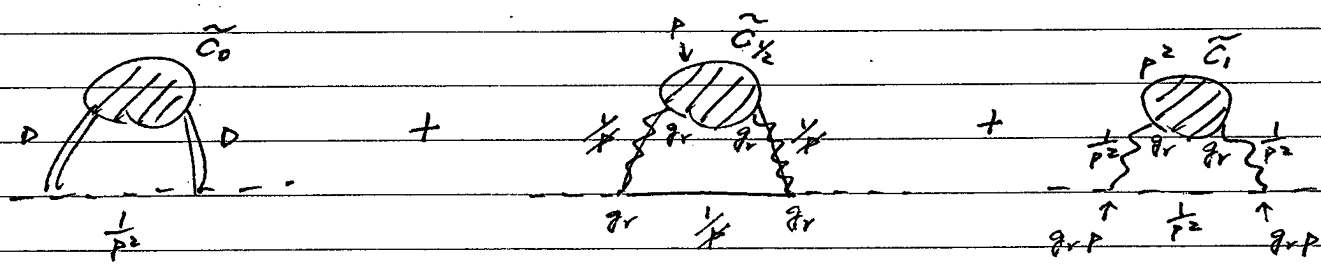
$$\boxed{m_\lambda = g^2 M \tilde{B}_{1/2}^{(r)}} \longleftarrow r = 1, 2, 3 \text{ for } U(1),\ SU(2),\ SU(3)$$

2° sfermion masses :



$\sim g_1^2 \, Y_f \, \zeta$

Y_f: The hyper charge for MSSM fields.



$$\sim g_r^4 \, C_2 \int d^4k \, \frac{1}{k^2} \left(\tilde{C}_{1/2} \, , \, \tilde{C}_1 \, , \, \tilde{C}_0 \right)$$

C_2: Casimir.

$$\therefore \quad \boxed{m_{\tilde{f}}^2 = g_1^2 Y_f \xi_1 + \sum_{r=1}^{3} g_r^4 \, C_2(f;r) \, A_r}$$

$$A_r = -\frac{M^2}{16\pi^2} \int dy \left(3\tilde{C}_1^{(r)}(y) - 4\tilde{C}_{1/2}^{(r)}(y) + \tilde{C}_0^{(r)}(y) \right)$$

$$y = k^2/M^2$$

$m_{\tilde{f}}^2 \to 0$ when $M^2 \to 0$

— The entire param-space of GM:

$$\underbrace{A_1,\ A_2,\ A_3}_{} \ ,\ \underbrace{B_1,\ B_2,\ B_3}_{}$$

Re, scalar mass B, gaugino mass.

assume $\xi = 0$.

$$\left.\begin{aligned} m_Q^2 &= A_3 + A_2 + \left(\tfrac{1}{6}\right)^2 A_1 \\ m_U^2 &= A_3 + \left(\tfrac{2}{3}\right)^2 A_1 \\ m_D^2 &= A_3 + \left(\tfrac{1}{3}\right)^2 A_1 \\ m_L^2 &= A_2 + \left(-\tfrac{1}{2}\right)^2 A_1 \\ m_E^2 &= A_1 \end{aligned}\right\}$$

5 m_{soft}'s defined by 3 param.

$\Downarrow$

2 constraints!

$$\boxed{\begin{aligned} &m_Q^2 - 2m_U^2 + m_D^2 - m_L^2 + m_E^2 = 0 \\ &2m_Q^2 - m_U^2 - m_D^2 - 2m_L^2 + m_E^2 = 0 \end{aligned}}$$

$$\Rightarrow \begin{cases} \mathrm{Tr}\, Y m_\phi^2 = 0 \\ \mathrm{Tr}\, (B-L)\, m_\phi^2 = 0 \end{cases}$$

Can also be derived by the $U(1)_Y$ & $U(1)_{B-L}$ anomaly free.

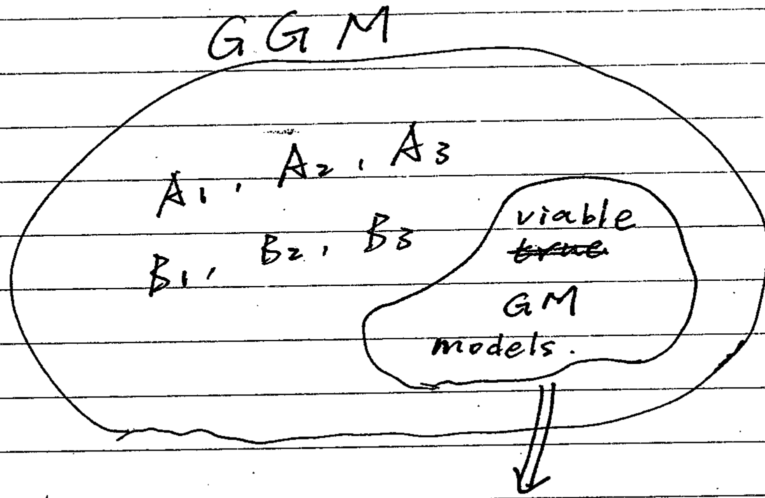
These relations are completely general features of GM, which do not depd on any specific form of the hidden of messenger sector.

Remark

1° The mass relation derived here is to $O(\alpha^2)$. One can show that, when imposing the messenger parity, the relation is correct to $O(\alpha^3)$.

2° The tadapole term $\propto Y_f$ is dangerous for tachyonic. One can impose the mess-parity to forbid $\langle J \rangle$.

— Does more sum rules exist?



Is it possible the viable GM models only span a subset of the GGM space?

$\Rightarrow$ Need to give an existence proof of the models that spans the whole GGM space.

(weakly coupled messenger model)

For $G = SU(3) \times SU(2) \times U(1)$, we have

$$A_k = \sum_R N_{k,R} A_R \,, \qquad B_k = \sum_R N_{k,R} B_R .$$

diff reps.

In order to cover the full param-space:

1° Need mess transforming in a least three diff reps.

2° Or can use 2 copies of the reps plus the D-type ~~SUSY~~.

$$\Rightarrow \quad M_B^2 = \begin{pmatrix} M_F^\dagger M_F + D & F \\ F & M_F M_F^\dagger + \bar{D} \end{pmatrix}$$

to ~~be~~ make the ~~mass~~ $(A_1, A_2, A_3, B_1, B_2, B_3)$ ~~more indp.~~

$\boxed{\text{Possible Solutions}}$ (simplest)

$$2 \times (10 \oplus \overline{10}) \quad \text{or} \quad 2 \times (5 + \bar{5}) \oplus 10 \oplus \overline{10}$$

This model covers the whole param.

$\Rightarrow$ $\boxed{\text{There cannot be any additional field theoretic restrictions on the GGM param space.}}$

Remarks: The gaugino & sfermion masses alone will ~~not be enough to distinguish diff GM scenarios~~