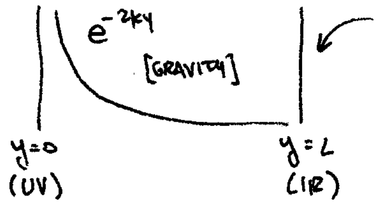
THE RANDALL SUNDRUM SCENARIO : what pheno students understand

5D, $ds^2 = \underbrace{e^{-2ky}}_{\text{WARP FACTOR}} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2$ $\left(= \left(\frac{R}{z}\right)^2 \left[dx^2 - dz^2\right]\right)$

metric of AdS

SOME PHENO : it's good for you (like bran) ... we'll get to strings

HIERARCHY PROBLEM : WHY is $M_{Pl} \gg M_W$?
RANDALL - SUNDRUM : ELECTROWEAK PHYSICS IS JUST HIGHLY REDSHIFTED

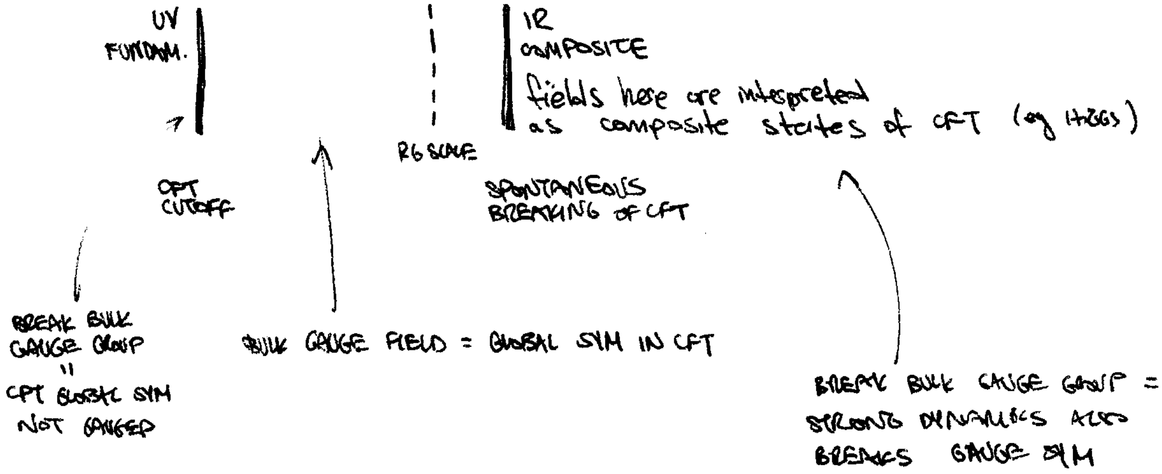


RS1 : SM FIELDS ARE 4D & LIVE ON THIS BRANE
[note: this is different from ADD, where $M_{Pl}^{5D} \sim$ TeV but $M_{Pl}^{4D} = 10^{19}$ GeV]

"modern" RS models have more bells & whistles
... bulk fields (Higgs still peaked on IR)
... can throw all sorts of things in there, eg penguins
but this shows the main features that we care about.

XD w/ AdS? THIS SOUNDS A LOT LIKE AdS/CFT.
IN FACT, I CAN PITCH THIS IN A WAY THAT EVEN EXPERIMENTALISTS CAN IDENTIFY WITH:

What are the collider signatures of XD? tower of KK resonances.
Expt.'list just says: ah! BUT WE'VE SEEN THAT ALREADY ... in '60s
⇒ HADRONIC RESONANCES (BOUND STATES)



fields here are interpreted as composite states of CFT (eg HIGGS)

BREAK BULK GAUGE GROUP
=
CFT GLOBAL SYM NOT GAUGED

BULK GAUGE FIELD = GLOBAL SYM IN CFT

BREAK BULK GAUGE GROUP = STRONG DYNAMICS ALSO BREAKS GAUGE SYM

Q : is THERE A STRING REALIZATION OF THIS WHERE CORRESPONDENCE is WELL-UNDERSTOOD? (YES) THEN WHAT STRING OBJECTS GENERATE THIS STRUCTURE?

START W/ WHAT WE KNOW

↳ AdS/CFT (Maldacena conjecture) $\quad \underset{\text{low E open strings}}{AdS_5 \times S^5} \longleftrightarrow \underset{\text{near horizon}}{\mathcal{N}=4 \text{ SCFT}}$

N D3s @ smooth point

thus we want to reduce the isometry group on this side $\Leftarrow$ way too much! Ideally $\mathcal{N}=0$ for RS, but at least $\mathcal{N}\leq 1$ for realistic χral TeV-scale model

KW (Klebanov-Witten) $\quad AdS_5 \times T^{1,1} \longleftrightarrow \mathcal{N}=1$ SCFT

PUT D3s @ SINGULAR POINT!
(OTHERWISE LOOKS LIKE ABOVE)

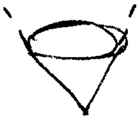
no RG flow ☺

$T^{1,1}$ $= SU(2)\times SU(2)/U(1)$
$\cong S^2 \times S^3$ (topologically)

WHAT IS THIS $T^{1,1}$?

RECALL THAT WE COMPACTIFY SUPERSTRING THY ON 6D CALABI-YAU 3-fold MANIFOLDS $(R_{MN}=0)$
(⟶ PRESERVES SOME SUBSET OF ORIGINAL 10D SUSY)

A 5D SASAKI-EINSTEIN SPACE X_5 IS DEFINED TO BE ~~THE~~ A MANIFOLD s.t. THE 6D CONE W/ X_5 AS ITS BASE IS A NON-COMPACT CALABI-YAU

 X_5 s.t.  = CY (🐟, KABELJAU)

$X_5 = T^{1,1}$ IS A PARTICULAR EXAMPLE WHOSE CONE IS CALLED THE CONIFOLD
(note that the conifold radius = AdS radius)

{ UNIMPORTANT NOTES

$$d\Omega^2_{T^{1,1}} = \frac{1}{9}(d\psi + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2)^2 + \frac{1}{6}(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) + \frac{1}{6}(d\theta_2^2 + \sin^2\theta_2 d\phi_2^2)$$

CONIFOLD: $ds^2 = dr^2 + r^2 d\Omega^2_{T^{1,1}}$

CONIFOLD: $\boxed{\sum_i^4 (z^i)^2 = 0}$ for $z \in \mathbb{C}^4$

THIS IS CLEARLY A CONE SINCE $z_i \to \lambda z_i$ IS STILL A SOLUTION

BUT THIS IS NONCOMPACT ?! INDEED. BY ITSELF THE CONIFOLD ISN'T REALISTIC, BUT IT TURNS OUT TO BE A CONVENIENT WAY TO UNDERSTAND THE IR BEHAVIOR (near-tip) OF MORE COMPLICATED CONFIGURATIONS WHERE THE CONIFOLD HAS A $\underbrace{\text{COMPACT UV COMPLETION}}_{\text{will be "UV brane"}}$. WE'LL GET TO THIS IN A BIT!

<u>CLAIM</u>: BASE OF THE CONIFOLD = $S^2 \times S^3$

Pf/ WRITE IN $\mathbb{R}$ COORDINATES $z^i = x^i + i y^i$

$$\sum_i (z^i)^2 = 0 = \sum_i (x^i)^2 - (y^i)^2 + 2i\, x^i y^i$$
$$= \underbrace{\vec{x}^2 - \vec{y}^2}_{} + \underbrace{2i\, \vec{x}\cdot\vec{y}}_{}$$

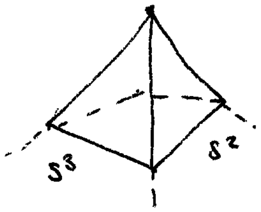
note: <u>squared</u>
not mod-squared!

~~[illegible]~~

$\vec{x}\cdot\vec{y} = 0$

$\vec{x}^2 = \rho^2 \longrightarrow S^3$ w/ RADIUS ρ

$\vec{y}^2 = \rho^2 \longrightarrow S^2$ fibered over S^3



where S^3 is RADIUS OF S^3
S^2 is RADIUS OF S^2

[see "better pic" below]

<u>THE POINT</u>: THIS IS PROMISING + INTERESTING BECAUSE WE HAVE A HANDLE ON BOTH THE GAUGE + GRAVITY SIDES.

<u>KT</u> (Klebanov-Tseytlin) $AdS \times T^{1,1}$ $\longleftrightarrow$ $\mathcal{N}=1$ SYM

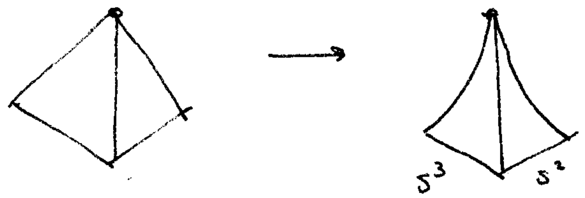
N D3s

new thing $\longrightarrow$ M D5s (wrapped)

<u>BREAK</u> CONFORMAL INV.
CAN BE UNDERSTOOD
VIA <u>DUALITY CASCADE</u>
W/ NO END!

We'll discuss this more in just a moment, but the idea is that in addition to the N D3s, include M D5 branes that wrap around the $S^2 \subset T^{1,1}$
$\Rightarrow$ "<u>FRACTIONAL D3</u> BRANES"

The BACKREACTION OF THESE FRAC. D3s GIVES WARPING
$\Downarrow$
<u>BREAKS</u> CONFORMALITY
& INTRODUCES <u>LOG</u> RUNNING.

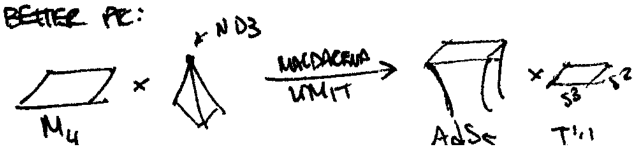


HEURISTICALLY, LOOK @ 't HOOFT COUPLING:

$$\lambda = g_{YM}^2 N \to g_{YM}^2 N(r)$$
$$\sim N + g_s M^2 \log(r/r_0)$$

LOOKS CLOSER TO RS... BUT $\overset{\text{NAKED}}{\wedge}$ CURVATURE SINGULARITY @ TIP.

BETTER PIC:

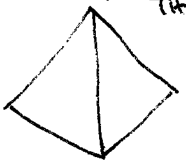


Maldacena: $g_s \ll 1$
$g_s N \gg 1$

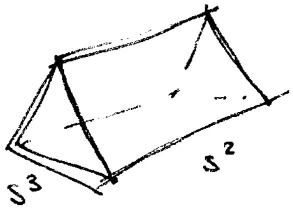
<u>KS</u> (Klebanov - Strassler)

bg-corr.
AdS × "$T^{1,1}$" ⟷ $\mathcal{N}=1$ SYM
N D3s — CASCADING TO
M D5s — A CONFINING
<u>DEFORMED</u> — THEORY IN IR

[forget warping for now]

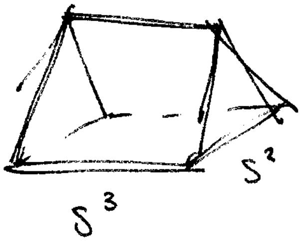


RESOLVED CONIFOLD →



BLOW UP S^2
... not interesting to us @ the moment

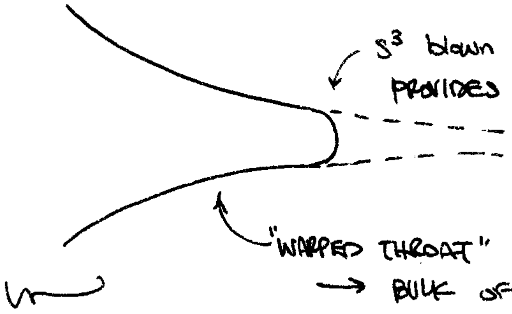
↓ <u>DEFORMED</u> CONIFOLD



BLOW UP S^3

$$\boxed{\sum_i (z^i)^2 = \epsilon^2}$$

SO:

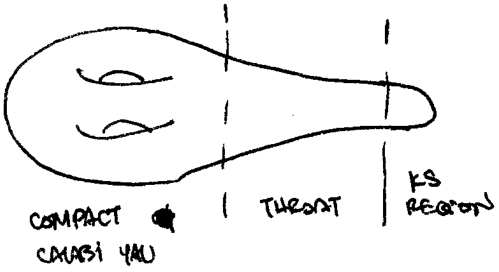


S^3 blown up w/ $r_{min} \approx \epsilon^{2/3}$
PROVIDES IR CUTOFF

"WARPED THROAT" REGION
→ BULK OF RS

WHAT ABOUT
THE UV CUTOFF?.

WANT TO STICK THIS ENTIRE CONSTRUCTION ON A COMPACT CY.
IE WANT TO EMBED IN A <u>FLUX COMPACTIFICATION</u>



<u>GKP</u>
Giddings
Polchinski
Kachru

ACTUALLY, THIS STEP IS PERHAPS THE MOST IMPORTANT SO WE'LL BE THE MOST DETAILED (or... THE LEAST HAND-WAVEY) FOR IT.

<u>HERE'S THE POINT</u>

GO BACK TO RS MODEL. WE CLAIMED WE GOT <u>EXPONENTIAL</u> WARPING OF IR SCALE RELATIVE TO UV SCALE. WHERE DID THE EXP COME FROM? $ds^2 \sim e^{-2ky} dx^2 + dy^2$

... BUT HOW DO I KNOW THAT THIS isn't JUST A MISLEADING CHOICE OF VARIABLES?

→ the important thing is <u>RADIUS STABILIZATION</u>
in RS, for eg, ∂ THE <u>GOLDBERGER-WISE MECHANISM</u>

IN STRING THEORY WE WANT TO PERFORM THE ANALOGOUS STABILIZATION OF THE WARPING ⇒ <u>MODULI STABILIZATION</u>

↓

FLUX COMPACTIFICATIONS

<u>RECALL</u>: INTERNAL MANIFOLD (CY) HAS MODULI - FIELDS w/ NO POTENTIAL CORRESPONDING, eg, TO THE SIZE (KÄHLER MODULI) & SHAPE (COMPLEX STRUCTURE MODULI) OF THE COMPACT SPACE.

WE WOULD LIKE TO FIX THESE MODULI - - - -> We are particularly interested in ϵ^2; deformation of conifold.

→ GIVE THEM A POTENTIAL (s.t. technically 'pseudomoduli')
↑ DYNAMICALLY GENERATED

This can be done by turning on the <u>field strengths</u> of some pform fields along the compact directions.
↑
"FLUX"

This is a big topic that I am not qualified to review.
<u>BUT</u>: it turns out that this is very closely related to WARPED COMPACTIFICATIONS. LET'S SEE HOW.

<u>COMMENTS</u>

- SEE BBS §9.6 (generally ch. 9-10) FOR MORE BG
- SEE ALSO SHAMIT'S REVIEWS ON FLUX COMPACTIFICATIONS
  eg ROLE OF FLUX FOR STABILIZING MODULI

GKP & FLUX COMPACTIFICATIONS IN IIB

HISTORICAL NOTE, CONVENTIONS, ROLE OF BRANES

$$S_{IIB} = \frac{1}{2\kappa^2}\int d^{10}x\,\sqrt{-G}\left[R - \frac{|\partial\tau|^2}{2(\mathrm{Im}\,\tau)^2} - \frac{|G_3|^2}{2\,\mathrm{Im}\,\tau} - \frac{|\tilde{F}_5|^2}{4}\right] + \frac{1}{8i\kappa^2}\int\frac{C_4\wedge G_3\wedge G_3^*}{\mathrm{Im}\,\tau}$$

only along compact space

$$G_3 = \underset{\underset{dC_2}{\uparrow}}{F_3} - \tau\,\underset{\underset{dB_2}{\uparrow}}{H_3} \qquad \tau = C_0 + ie^{-\Phi}$$

axion-dilaton ; "HOLOMORPHIC COUPLING"

$\tilde{F}_5$ is the self-dual 5-form $\leftarrow$ $= F_5 - \frac{1}{2}C_2\wedge H_3 + \frac{1}{2}B_2\wedge F_3$

$= (1+*_{10})\, d\alpha\wedge dx^0\wedge\ldots\wedge dx^3$

for some $\alpha(y)$

PUNCHLINE: EINSTEIN EQ:

$$R_{MN} = \kappa^2\left(T_{MN} - \frac{1}{8}\, g_{MN}\, T\right)$$

$\uparrow$ ANSATZ: $ds_{10}^2 = e^{2A(y)}\eta_{\mu\nu}\,dx^\mu dx^\nu + e^{-2A}dy^2$

$\downarrow$ trace + ansatz

$$(\nabla_6)^2 e^{4A(y)} = \underbrace{\frac{e^{2A}|G_3|^2}{12\,\mathrm{Im}\,\tau} + e^{-6A}\left[|\partial\alpha|^2 + |\partial e^{4A}|^2\right]}_{\text{POSITIVE DEFINITE! } > 0} + \underbrace{\frac{\kappa^2}{2}e^{2A}\left(T^M{}_M\right)^{loc}}_{\text{BRANE TERMS.}}$$

"change in warp factor"

TOTAL DERIVATIVE

$\downarrow$

INTEGRATES TO ZERO

IN THE 80'S PEOPLE DIDN'T PROPERLY UNDERSTAND ROLE OF LOCALIZED SOURCES (BRANES), SO THIS WAS TAKEN AS A NO-GO THM FOR WARPED COMPACTIFICATIONS.

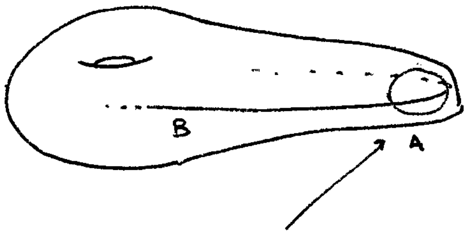
NOW WE KNOW THAT WE JUST NEED NEGATIVE TENSION LOCALIZED OBJECTS.

$\rightarrow$ negative tension branes
(TURNS OUT TO BE GENERIC ON CY)

$\}$

WE CAN SEE THIS ALREADY IN RS
SOLVING EINSTEIN EQ GIVES REQ. THAT
UV + IR BRANE TENSIONS ARE NEGATIVE.

ONWARDS TO GKP



what is size of A?

WE CAN DEFINE A BASIS OF $\overset{\text{DUAL}}{3}$ CYCLES ON OUR CY w/ A THROAT (KS)

A: S^3 OF $T^{1,1}$ @ TIP
B: S^2 × RADIAL AdS DIRECTION

(GENERALLY $h^{2,1}$ COPIES OF A,B BASIS)
↑ HODGE # OF MANIFOLD

(has to do w/ $\mathbb{C}$ STRUCTURE MODULI SPACE)

RECALL: M fractional D3s $\longrightarrow$ $\int_A F_3 = 4\pi^2\alpha' M$
N D3s $\longrightarrow$ $\int_B H_3 = -4\pi^2\alpha' K$

$$\underset{\substack{\uparrow \\ N = MK}}{K}$$

{ <u>sanity check</u>: D3 charge conservation / Bianchi Identity (BBS 10.136)

$$\underset{\substack{\uparrow \\ \int_{M_6} d\tilde{F}_5}}{\int_{M_6} H_3 \wedge F_3} = 2K^2 T_3 \underbrace{(MK)}_{N}$$

<u>FACT</u>: $W = \int_{M_6} G_3 \wedge \Omega$

↑ nowhere vanishing HOLOMORPHIC 3-form [CY n-fold has n-form Ω]

↓

$$= 4\pi^2\alpha'\left(M \underbrace{\int_B \Omega}_{\substack{\uparrow \\ \frac{z}{2\pi i}\log(z) + \text{regular}}} - K\tau \underbrace{\int_A \Omega}_{z}\right)$$

z ~ SIZE OF A-CYCLE; what we want

$\langle\tau\rangle = \frac{\theta}{2\pi} + \frac{i}{g_s}$

MIN OF SCALAR POT: $D_z W = 0 = 4\pi^2\alpha'\left(\frac{M}{2\pi i}\log z - i\frac{K}{g_s} + \cdots\right)$

$$\Rightarrow \ln z = -\frac{2\pi K}{M g_s} \Rightarrow \boxed{z \simeq e^{-\frac{2\pi K}{M g_s}}}$$

in the limit K/g_s large (eg $M=1$, $K/g_s = 5$)
(gives large hierarchy!)

z IS LIKE VOL OF S^3 → $\underset{\text{AdS WORD}}{r_{\min}} \sim z^{1/3}$ ✓

WE'VE THUS FOUND SOMETHING RS-LIKE WARPED 'MACROSCOPIC' THROAT W/ $\overset{\text{MICROSCOPIC}}{\text{STRINGY}}$ STRUCTURE CUTTING IT OFF IN THE UV + IR (which phenomenologists call branes)

BUT THERE IS A VERY INTERESTING ELEPHANT IN THE ROOM:

WHAT IS THE GAUGE DUAL? FIRST IGNORE $\overset{\text{WRAPPED}}{\text{D5}}$ BRANES

FACT: ORBIFOLDING THE S^5 IN $AdS_5 \times S^5$ GIVES A PRODUCT GAUGE GROUP namely, $S^5/\mathbb{Z}_2 \longrightarrow SU(N) \times SU(N)$ W/ BIFUNDAMENTAL S

FACT: $T^{1,1}$ CAN BE OBTAINED FROM SMOOTHING OUT $S^5/\mathbb{Z}_2$
→ DEDUCE $SU(N) \times SU(2)$ W/ BIFUNDAMENTALS

	$SU(N)$ ×	$SU(N)$ ×	$SU(2)$ ×	$SU(2)$ ×	$U(1)_R$
A	$\square$	$\overline{\square}$	$\square$	1	1/2
B	$\overline{\square}$	$\square$	1	$\square$	1/2

from $T^{1,1} \sim SU(2)^2/U(1)$

$$W \sim \underset{r,u}{\mathrm{Tr}\,\mathrm{Det}}\,(A_r B_u) = A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1$$

$\uparrow \underset{r \quad\; u}{SU(2) \times SU(2)}$ W/ GAUGE INDICES CONTRACTED

IF WE INCLUDE THE M $\overset{\text{WRAPPED}}{\text{D5's}}$, GAUGE GROUP BECOMES $SU(N+M) \times SU(N)$ IN THE UV, ie $r > r_{min}$ (where WRAPPED D5's FALL TO r_{min})

OUR LOG RUNNING GIVES:

$$\underbrace{\frac{1}{\alpha_1(\mu)}}_{SU(M+N)} + \underbrace{\frac{1}{\alpha_2(\mu)}}_{SU(N)} = \frac{1}{g_s}$$

$\downarrow$

$$\frac{1}{\alpha_1(\mu)} - \frac{1}{\alpha_2(\mu)} \sim \log\frac{\mu}{\Lambda} + \text{const}$$

$\boxed{\text{SO}}$: AS WE DECREASE μ: α_1 INCREASES
α_2 DECREASES

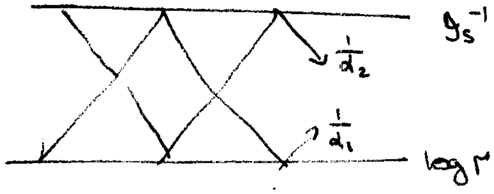
→ EVENTUALLY $1/\alpha_1 \to 0$

SEIBERG DUALITY

$SU(C)$ w/ $F > C$ FLAVORS

$$\Big\downarrow \text{ seiberg dual}$$

$SU(F-C)$ w/ F FLAVORS



THUS: $\overbrace{SU(\underbrace{M+N}_{C})}^{\alpha_1} \times \underbrace{SU(N)}_{\Rightarrow F = 2N}$ $\quad (SU(2) \ne SU(N))$

$$\downarrow$$

$SU(N-M) \times SU(N)$

$$\downarrow$$

$SU(N-M) \times SU(N-2M)$

$\boxed{\text{DUALITY CASCADE}}$

$\rightsquigarrow$ if $N = kM$, this happens k times
(k acts as a 'clock')

$SU(M)$ w/ no chiral matter

$\nwarrow$ $SU(M) \times \underline{SU(0)}$ = no matter

$\uparrow$ this is the IR theory

OF CSABA'S COURSE, WE KNOW!

$b_0 = 3M \longrightarrow \Lambda^{3M} = \mu^{3M} e^{-8\pi^2/g^2}$

this has a discrete spectrum w/ <u>mass gap</u>, confinement

$$U(1)_R \xrightarrow{\text{anomaly}} \mathbb{Z}_{2M} \xrightarrow[\text{GAUGINO CONDENS}]{\langle\lambda\lambda\rangle} \mathbb{Z}_2$$