

HISTORICAL CONTEXT (ASIDE) \rightarrow E&M, CAN CAST E&M IN PSEUDOTANGENTIAL FORMALISM; WE'LL DO THIS LATER.

BY END OF 1800's, PHYSICISTS THOUGHT THEY WERE "DONE"
E&M + MECHANICS (i.e. GRAVITY) SEEMED TO EXPLAIN EVERYTHING!

... EXCEPT:

- WHY DOES THE SUN SHINE? (NUCLEAR PHYS.)
- WHY IS THE SKY BLUE? (maybe)
- WHY ARE SOME MATERIALS CONDUCTORS/INSULATORS? (QM)
- WHY DO ALL ELECTRONS HAVE THE SAME CHARGE? (QFT)
- WHAT DETERMINES THE THERMODYNAMIC PROPERTIES OF MATERIALS? (STAT M)
- WHY ARE THERE 3+1 DIMENSIONS? (STRING/M WHEEL)

CAN EVEN ADD
SPECIAL RELATIVITY!

SO WHY DID SCIENTISTS THINK THEY WERE DONE?

- \rightarrow CM DESCRIBED THINGS ON SCALES WE UNDERSTOOD
- \rightarrow THIS IS THE IDEA OF "EFFECTIVE (FIELD) THEORY"
- PHYSICS @ DIFFERENT SCALES (ENERGY, LENGTH, etc.)
SHOULDN'T SIGNIFICANTLY AFFECT PHYSICS AT THE
SCALE ONE IS MEASURING.

i.e. "A CHEF DOES NOT NEED QFT" - STUAS DIMOPOLOUS.

- THIS IS WHY THE LHC IS EXCITING
IT WILL PROBE NEW ENERGY SCALES THAT
MAY HAVE EXCITING NEW PHYSICS

EVEN TODAY IT LOOKS LIKE PHYSICISTS ARE "DONE"

C WELL, 1970's

- \rightarrow THE "STANDARD MODEL" IS SELF CONSISTENT
("RENORMALIZABLE" IF YOU WANT TO BE FANCY-SCHMANCE)
- \rightarrow BUT STILL LOTS OF QUESTIONS TO BE ASKED & ANSWERED

HOW THIS CAN BE SEEN IN E&M (HEURISTIC!)
(FROM H NAKAYAMA)

LET US MODEL THE COULOMB SELF-ENERGY OF THE ELECTRON BY

$$E_c = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r_e} \quad \text{WHERE } r_e \text{ IS AN EFFECTIVE ELECTRON RADIUS}$$

(UP TO SOME NUMERICAL PREFACCTOR OF ORDER 1
DEPENDING ON WHETHER WE MODEL THE e^- AS A
SOLID BALL OR A SHELL)

These notes are
written by the
lecturer to help
you understand
the curriculum.

NOW, WE "KNOW" THAT THE REST ENERGY OF THE ELECTRON IS $(M_ec^2)_{\text{obs}}$.
 I WILL MARK THIS AS THE OBSERVED ELECTRON REST ENERGY.
 WHY? TO DISTINGUISH IT FROM THE "BARE" REST ENERGY $(M_ec^2)_B$ THAT WE WRITE IN OUR EQUATIONS.

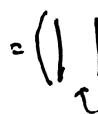
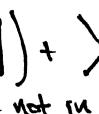
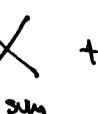
→ THIS IS A WEIRD THING TO SAY
 IN QFT, FEYNMAN DIAGRAMS ARE A TAYLOR EXPANSION OF PROBABILITY AMPLITUDES

e.g.  is AN AMPLITUDE FOR 2 PARTICLES GOING IN + 2 PARTICLES GOING OUT

OUR THEORY (LAGRANGIAN) DETERMINES THE KINDS OF VERTICES (INTERACTIONS WE CAN DRAW).

i.e. LET'S SAY 2 PARTICLES CAN HAVE A 4-PT INTERACTION :



THEN  =  +  +  +  + ...
 { NOT IN SUM }

HOWEVER, THE MASSES/COPPLINGS THAT GO INTO EACH FEYNMAN DIAGRAM ARE DIFF FROM THE OBSERVABLE MASSES/COPPLINGS IN A PHYSICAL PROCESS, WHICH IS THE SUM OF DIAGRAMS.

i.e. $(\text{X})_{\text{Physical}} = \text{X} = \text{X} + \text{X} + \text{X} + \dots$

[IF YOU'RE CONFUSED, DON'T WORRY!!]

ANYWAY, THE OBSERVED REST ENERGY SHOULD BE THE SUM OF THE BARE REST ENERGY w/ THE COULOMB SELF ENERGY:

$$\begin{aligned}(M_ec^2)_{\text{obs}} &= (M_ec^2)_B + E_c \\ &= (M_ec^2)_B + \frac{1}{4\pi\epsilon_0} \frac{e^2}{r_e}\end{aligned}$$

EXPERIMENT TELLS US $r_e \lesssim 10^{-17}$ cm

$$\Rightarrow E_c \gtrsim 10 \text{ GeV}$$

$$\text{BUT } (M_ec^2)_{\text{obs}} = 0.511 \text{ MeV} !$$

Why is this weird? We get a large "fine-tuning"

$$0.511 = -9999.489 + 10000.000$$

↑ ↑ {
 OBS BARE REST E ~E_c
 is NEGATIVE
 (it's ok - it's just a mathematical #)

ISN'T IT ODD THAT NATURE CARES ABOUT 5 DECIMAL POINTS?
 WHAT A "MIRACULOUS" CANCELLATION!

ALTERNATE POINT OF VIEW

FINE TUNING IS A HINT OF NEW PHYSICS
 SUPPOSE THERE IS NO FINE TUNING,
 THEN $E_c = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r_e}$

MUST BE INVALID ON SCALES SHORTER THAN
 $r \sim \frac{e^2}{4\pi\epsilon_0 (m_e c^2)} \approx 2.8 \times 10^{-13} \text{ cm}$

THE RESOLUTION: 3 POSITRON!

NEVERMIND THE MOTIVATION FOR THE POSITRON,
 BUT YOU KNOW FROM PHYS TO THAT EM FIELD IS
 "MADE UP OF" PHOTONS (γ) THAT CAN PAIR PRODUCE:
 $T \rightarrow e^+ e^-$

ALSO FROM PHYS TO $\Delta t \Delta E \sim \hbar$
 FOR QUANTUM EFFECTS

THUS WE EXPECT PAIR PRODUCTION TO BECOME RELEVANT
 AT DISTANCE SCALES OF

$$d \sim c \Delta t \sim \frac{\hbar c}{(2m_e c^2)} \approx 200 \times 10^{-13} \text{ cm}$$

\Rightarrow CLASSICAL EM BREAKS DOWN AT AROUND THE SCALE
 WE PREDICTED!

EVEN MORE DETAIL

SO OUR PICTURE IS:

- ON SCALES $\gg 10^{-11}$ cm : USUAL POTENTIAL
- ON SCALES $\approx 10^{-11}$ cm : VIRTUAL PAIRS OF e^+ e^-
MATERIALIZE OUT OF THE EM FIELD
TO SCREEN THE $1/r$ POTENTIAL

V. WEISSKOPF (1939) WAS THE 1ST TO CALCULATE THE CONTRIBUTION TO THE e^- SELF-ENERGY FROM PAIR PRODUCTION:

$$E_{PAIR} = -\frac{1}{4\pi\epsilon_0} \frac{e^2}{r_e} \quad |_{r \approx 10^{-11}\text{cm}}$$

↓ MINUS SIGN! → CANCELS E_c TO LEADING ORDER

$$\text{SO } \Delta E = E_c + E_P$$

AS $r_e \rightarrow 0$, $\Delta E \rightarrow \frac{3\epsilon}{4\pi} m_e c^2 \log \frac{r}{r_{BARE}}$

$$(m_e c^2)_{\text{OBS}} = (m_e c^2)_{\text{BARE}} \left[1 + \frac{3\epsilon}{4\pi} \log \frac{r}{m_e c r} \right]$$