AGENDA

- ANNOUNCEMENTS
- EM WAVES – the nitty gritty
  - reflection + transmission (SNELL'S LAW)
  - absorption & dispersion
  - other details
- PROBLEMS
- PHASE v. GROUP
- EXTRA TOPICS

ANNOUNCEMENTS

- PLEASE OBSERVE THE HW POLICY & HONOR CODE (SOME QUESTIONABLE CASES IN HW #1, WE WILL BE MORE STRICT IN THE FUTURE.)
- MAGNETIC MONOPOLE (+ 2 PHOTONS) PAPER ON COURSEWORK ref:

EM WAVES – REVIEW

"sanity": what happens in MEDIA? $c \to v$

$$\boxed{\begin{aligned} \tilde{E}(\vec{r},t) &= \tilde{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}\,\hat{n} \\ \tilde{B}(\vec{r},t) &= \tfrac{1}{c}\tilde{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}\,\hat{k}\times\hat{n} = \tfrac{1}{c}\hat{k}\times\tilde{E} \end{aligned}}$$

(9.49) VAC.

sanity check: what does the tilde mean? how to write in $\mathbb{R}$ notation?

YOU ALSO KNOW SOME EXPRESSIONS FOR $E, \vec{P}$
eg: u, $\vec{S}$, $\wp$, I, etc. & AVERAGES

YOU DID PROBLEM 9.6 ON HW #2 re: WAVES ON A STRING WITH A MASSIVE KNOT.

Lessons: ① PHYSICS OF WAVES w/ BC
② BC ⇒ SOLUTIONS OF WAVE EQ.

Now things get interesting! (i.e. UGLIER)

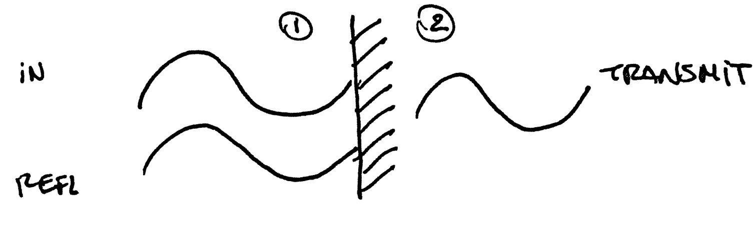
EM WAVES → OPTICS

my personal recurring theme in E&M: YOU FINALLY UNDERSTAND SOMETHING REASONABLY WELL & THEN THE CLASS HAS TO MUCK IT ALL UP BY INTRODUCING MEDIA !!

CAUTION:
1. REPEAT THE MANTRA OF PHYSICS VS. MATH
2. TRICKY PT. IS HOLDING ON TO AN UNDERSTANDING OF PHYS AS YOU SLOG THROUGH THE MATH !

→ THE RESULT IS SURPRISING (SNELL'S LAW)

HEURISTIC PICTURE



Same story
- STRING w/ KNOT
- EM WAVE $\perp$
- EM @ OBLIQUE $\measuredangle$
- QM 'WELL POTENTIAL'
- MOST PDE'S

GENERAL FORM OF WAVE EQ. SOL. | BOUNDARY CONDITIONS CONSTRAIN "REFL" & "TRANSMIT" IN TERMS OF "IN" | GENERAL FORM OF WAVE EQ. SOL

ASSUMPTIONS: $\rho_F,\ J_F = 0$

BC FOR EM IN MEDIA (recall ch. 7)

"POOR MAN'S GAUSS' LAW"

$$\boxed{\begin{array}{ll} \epsilon_1 E_1^{\perp} = \epsilon_2 E_2^{\perp} & \text{FROM GAUSS' LAW} \\ B_1^{\perp} = B_2^{\perp} & \text{FROM } \nabla\cdot\vec{B}=0 \text{ (i.e. MAGNETIC GAUSS' LAW)} \\ \vec{E}_1^{\parallel} = \vec{E}_2^{\parallel} & \text{FROM } \nabla\times\vec{E} = -\dot{\vec{B}} \text{ w/ AMPERIAN LOOP (LIM AREA} \to 0) \\ \frac{1}{\mu_1}\vec{B}_1^{\parallel} = \frac{1}{\mu_2}\vec{B}_2^{\parallel} & \text{FROM AMPERE'S LAW w/ NO FREE CURRENT} \end{array}}$$

(9.74)

IN CASE YOU DIDN'T REALIZE, THESE EQUATIONS ARE IMPORTANT! THIS IS WHERE SNELL'S LAW COMES FROM
→ note: nothing mysterious about the origins of these BC!

REFLECTION & TRANSMISSION @ OBLIQUE INCIDENCE

($\perp$ CASE IS UNINTERESTING/EASY)

I'M NOT GOING TO RE-DERIVE IT FOR YOU (that's like asking someone to describe a root canal they recently had!!)

WHAT GRIFFITHS DID (& WHY IT SHOULD BE FAMILIAR)

1. WROTE GENERAL FORMS FOR $\tilde{E}_I$, $\tilde{E}_R$, $\tilde{E}_T$ & $\tilde{B}$'s (9.49)
   → NOTE FREQ is FIXED!

   but $\omega = k_I v_1 = k_R v_1 = k_T v_2$ ← remember from last sec?
   ⇒ RELATES k's.

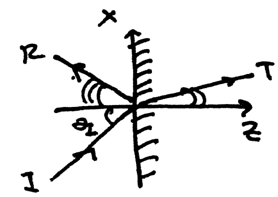
2. JOIN (I+R) FIELDS w/ (T) FIELD (CONTINUITY)

$$\_\_\, e^{i(\vec{k}_I\cdot\vec{r} - \omega t)} + \_\_\, e^{i(\vec{k}_R\cdot\vec{r} - \omega t)} = \_\_\, e^{i(\vec{k}_T\cdot\vec{r} - \omega t)} \quad @\, z=0$$

$\forall\ \vec{r}|_{z=0},\ t$ ⇒ exp's must be equal! (@ $z=0$)
⇒ $(\vec{k}_I)_\perp = (\vec{k}_R)_\perp = (\vec{k}_T)_\perp$ ⇐ $\perp$ MEANS (x,y) ONLY
(b/c $(\vec{k}\cdot\vec{r} - \omega t) = (-''-)|_{z=0}$)

PHYSICS: PLANE OF INCIDENCE ($\vec{k}$'s COPLANAR w/ $\hat{z}$)

2b) this also gives $\begin{cases} \theta_I = \theta_R \\ n_2 \sin\theta_T = n_1 \sin\theta_I \end{cases}$



↑ from $(k_I)_x = (k_R)_x = (k_T)_x$

$k_I \sin\theta_I = k_R \sin\theta_R = k_T \sin\theta_T$

$k_I = k_R$ since $\omega = k_I v_1 = k_R v_1$

$n = \frac{c}{v}$

③ NOW THE EXP'S CANCEL IN OUR EQ
APPLY BC FOR EM IN MEDIA (9.74)

also note: POLZ ∥ PLANE

→ WRITE EXPLICITLY $\perp \to z$
$\parallel \to x, y$ (PAIR OF EQ'S)

those will involve θ's
→ USE PT ② TO SIMPLIFY

⇒ ALGEBRA

$$\boxed{\tilde{E}_{0R} = \left(\frac{\alpha - \beta}{\alpha + \beta}\right)\tilde{E}_{0I}, \quad \tilde{E}_{0T} = \left(\frac{2}{\alpha + \beta}\right)\tilde{E}_{0I}}$$

FRESNEL EQ. (9.109) (POLZ ∥ PLANE)

$\alpha = \cos\theta_T / \cos\theta_I$
$\beta = \mu_1 v_1 / \mu_2 v_2$

note: $\alpha = \beta$ ⇒ BREWSTER'S ANGLE; no REFL.

Lesson: THERE'S WHERE OPTICS COMES FROM
also: BACK IN MY DAY, WE HAD TO DO 9.16
WHICH WAS REDERIVING FOR POLZ ⊥ PLANE.

ABSORPTION & DISPERSION - (CONDUCTORS)

WHAT HAPPENS TO BC WHEN $\rho_f, \vec{J}_f \neq 0$?

$$\boxed{\vec{J}_f = \sigma \vec{E}}$$

RECALL BC CAME FROM MAXWELL'S EQ.
SO LET'S START FROM THERE AGAIN & PLUG $\vec{J}_f = \sigma\vec{E}$

(i) $\nabla\cdot\vec{E} = \frac{1}{\epsilon}\rho_f$
(ii) $\nabla\cdot\vec{B} = 0$
(iii) $\nabla\times\vec{E} = -\dot{\vec{B}}$
(iv) $\nabla\times\vec{B} = \mu\sigma\vec{E} + \mu\epsilon\dot{\vec{E}}$

★

CONTINUITY: $\nabla\cdot\vec{J}_f = -\frac{\partial\rho_f}{\partial t}$

GAUSS: $\sigma(\nabla\cdot\vec{E}) = -\frac{\sigma}{\epsilon}\rho_f$

$\Rightarrow \rho_f(t) = e^{-(\sigma/\epsilon)t}\rho_f(0)$

$\tau \equiv \epsilon/\sigma$
CHARACTERISTIC TIME FOR DISSIP. OF FREE CHARGE.
(as expected!)

Q. what happens to free charge in a conductor?

FOR $t \gg \tau$, $\rho_f = 0$. (i) $\rightarrow \nabla\cdot\vec{E} = 0$
THIS WAS THE SAME AS NONCONDUCTING MEDIA,
except FACTOR OF $\mu\sigma\vec{E}$ IN (iv)

$$\Rightarrow \nabla^2\vec{E} = \mu\epsilon\ddot{\vec{E}} + \mu\sigma\dot{\vec{E}}$$
$$\nabla^2\vec{B} = \mu\epsilon\ddot{\vec{B}} + \mu\sigma\dot{\vec{B}}$$

I DON'T KNOW WHAT THESE TERMS ARE CALLED IN PDE MATHEMATICS, BUT I CALL THEM STUPID HEADACHES.
(not present in nonconducting case)

GEN SOLUTIONS

$\Rightarrow$ new eq's have same form, only COMPLEX k (as you'd expect)

$$\tilde{k}^2 = \mu\epsilon\omega^2 + i\mu\sigma\omega$$
$$\tilde{k} = k + i\kappa$$

$$k/\kappa = \omega\sqrt{\frac{\epsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2} \pm 1\right]^{1/2}$$

behaves as usual

this i GIVES EXP DAMPING

9.130 9.131
$$\Rightarrow \boxed{\begin{aligned}\tilde{\vec{E}}(z,t) &= \tilde{E}_0 e^{-\kappa z} e^{i(kz-\omega t)}\,\hat{x}\\ \tilde{\vec{B}}(z,t) &= \frac{1}{\omega}\tilde{k}\tilde{E}_0 e^{-\kappa z}e^{i(kz-\omega t)}\,\hat{y}\end{aligned}}$$

from $\nabla\times\vec{E} = -\dot{\vec{B}}$

note: $\tilde{k}$ now gives extra phase!

$$B_0 e^{i\delta_B} = \frac{|\tilde{k}|}{\omega}E_0 e^{i\delta_E}e^{i\phi}$$

INTERLUDE - AN EXAMPLE PROBLEM

GRIFFITHS 9.20

eq. 9.138 PLANE WAVES IN A CONDUCTOR

$$\begin{cases} \vec{E}(z,t) = E_0 e^{-\kappa z} \cos(kz - \omega t + \delta_E)\hat{x} \\ \vec{B}(z,t) = B_0 e^{-\kappa z} \cos(kz - \omega t + \delta_E + \phi)\hat{y} \end{cases}$$

(OR USE 9.130 - .131 ABOVE)

? $\langle u \rangle$. SHOW MAG. CONTR DOMINATES

IDEA : $\langle \cos^2 \rangle = \frac{1}{2}$

$$u = \frac{1}{2}\left(\epsilon E^2 + \frac{1}{\mu}B^2\right)$$

USE exp. for k/κ

$$u = \frac{1}{2}\left(\epsilon E_0^2 e^{-2\kappa z}\cos^2 \_\_ + \frac{1}{\mu}B_0^2 e^{-2\kappa z}\cos^2 \_\_\right)$$

TIME AVERAGE to 1/2

$$\langle u \rangle = \frac{1}{4}e^{-2\kappa z}\left(\epsilon E_0^2 + \frac{1}{\mu}\left(\frac{1}{\omega}|\tilde{k}|E_0\right)^2\right) \quad |\tilde{k}|^2 = \omega^2 \frac{\epsilon\mu}{2}\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2}$$

$$= \frac{1}{4}e^{-2\kappa z}\left(\epsilon E_0^2 + \frac{1}{\mu}\left(\frac{\epsilon\mu}{2}\sqrt{1-\left(\frac{\sigma}{\epsilon\omega}\right)^2}\right)E_0^2\right)$$

$$= \frac{1}{4}e^{-2\kappa z}\epsilon E_0^2\left(1 + \sqrt{1+(\sigma/\epsilon\omega)^2}\right)$$

E CONTR $\quad$ $\boxed{\text{B CONTR} , > 1}$ $\quad$ learn to see physics in the math!

$$= \frac{1}{4}e^{-2\kappa z}\epsilon E_0^2\left(\frac{k^2}{\omega^2}\cdot\frac{2}{\epsilon\mu}\right)$$

USING SOL. OF $\tilde{k}^2 = \mu\epsilon\omega^2 + i\mu\sigma\omega$

CONCEPTUAL : GRIF 9.18

(a) EMBED ρ_f ON GLASS, est. TIME to FLOW to SURFACE
ANSW: WANT CHARACTERISTIC TIME $\tau = \epsilon/\sigma = \left(\frac{\epsilon}{\epsilon_0}\right)\epsilon_0/\sigma$ (n^2)

(b) DESIGNING A μ-WAVE EXPT @ SOME MW FREQ. HOW THICK TO MAKE SILVER?
ANSW: SKIN DEPTH : $d \equiv 1/\kappa$ (ANALOGOUS TO CHAR TIME)

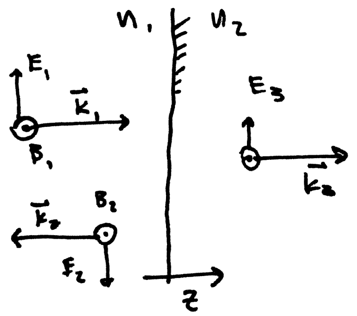
PROBLEM from

PROBLEMS & SOLUTIONS ON ELECTROMAGNETISM
Ed. LIM YUNG-KUO WORLD SCIENTIFIC
LIBRARY: QC 760.52 P76 1993 (RES.)

POLARIZ. PLANE EM WAVE IN MEDIUM OF INDEX n, REFLECTS @ NORMAL INCIDENCE FROM SURFACE OF A CONDUCTOR

(?) PHASE CHANGE IF CONDUCTOR HAS $n_2 = n_1(1+i\rho)$

SANITY: UNDERSTAND $i\rho$?



why are the REFL ARROWS AS THEY ARE? $v_2 < v_1$

$n = \frac{c}{v}$ $\quad |n_2| > |n_1|$
$\quad |v_2| < |v_1|$

$$\vec{E}_I = \vec{E}_{I0}\, e^{i(k_I z - \omega t)} \qquad \vec{B}_I = \vec{B}_{I0}\, e^{i(k_I z - \omega t)}$$

$B_{I0} = \frac{n}{c} E_{I0}$ ← RECALL IN VAC $B_0 = \frac{1}{c} E_0$

$$\vec{E}_R = \vec{E}_{R0}\, e^{i(-k_R z - \omega t)}$$

note $K_R = K_I = \frac{\omega}{c}$
(SAME MEDIUM)

$$\vec{E}_T = \vec{E}_{T0}\, e^{i(k_3 z - \omega t)}$$

BC: CONTINUOUS: $E_{10} - E_{R0} = E_{30}$

H CONTINUOUS

$B_{10} + B_{20} = \mu_1/\mu_2\, B_{30} \approx B_{30}$ nonferrom

$\Rightarrow E_{10} + E_{R0} = \frac{n_2}{n} E_{30}$

ALGEBRA: $E_{10} + E_{R0} = \frac{n_2}{n}(E_{10} - E_{R0})$

$E_{R0}\left(1 + \frac{n_2}{n}\right) = E_{10}\left(\frac{n_2}{n} - 1\right)$ $\quad a+bi$

$E_{R0} = \left(\frac{n_2 - n}{n_2 + n}\right) E_{I0} \Rightarrow \boxed{\varphi = \arctan \frac{b}{a}}$

CRUNCHING #'s: $\frac{n_2 - n}{n_2 + n} = \frac{\rho}{\sqrt{\rho^2 + 4}}\, e^{i\varphi}$

$$\varphi = \arctan\left(\frac{2}{\rho}\right)$$

GROUP & PHASE VELOCITY

GRIFFITHS REFERENCE CH. 9 # 13 P. 399
AP FRENCH : Vibrations & Waves

$$f_1 = A \sin(k_1 x - \omega_1 t) \qquad v_1 = \omega_1/k_1$$
$$f_2 = A \sin(k_2 x - \omega_2 t) \qquad v_2 = \omega_2/k_2$$

last section we wrote $f = A\sin(k(x - vt))$

$$f = f_1 + f_2$$
$$= A[\sin(k_1 x - \omega_1 t) + \sin(k_2 x - \omega_2 t)]$$

IDENTITY: $\sin(\theta + \varphi) + \sin(\theta - \varphi) = 2\sin\theta\cos\varphi$

$$\theta + \varphi = k_1 x - \omega_1 t$$
$$\theta - \varphi = k_2 x - \omega_2 t$$

$$f = 2A \sin\left(\tfrac{1}{2}(k_1 + k_2)x - \tfrac{1}{2}(\omega_1 + \omega_2)t\right) \cos\left(\tfrac{1}{2}(k_1 - k_2)x - \tfrac{1}{2}(\omega_1 - \omega_2)t\right)$$

lo freq
BIG λ

Hi freq
SMALL λ

think of as SHORT λ (RAPID) WAVE MODULATED BY LONG λ WAVE MOVING @ DIFF SPEEDS.

$$f = 2A \sin(kx - \omega t)\cos(\Delta k\, x - \Delta\omega\, t)$$

$v_{PHASE} = \omega/k$ $\qquad$ $v_{GROUP} = \Delta\omega/\Delta k = d\omega/dk$

ENVELOPE

$$k = \tfrac{1}{2}(k_1 + k_2) \qquad \Delta k = \tfrac{1}{2}(k_1 - k_2)$$
$$\omega = \tfrac{1}{2}(\omega_1 + \omega_2) \qquad \Delta\omega = \tfrac{1}{2}(\omega_1 - \omega_2)$$

GROUP VELOCITY : ENERGY (& INFO) TRANSPORT
(Physical stuff)

PHASE VELOCITY: MAY > c , BUT NO INFO TRAVELS.