

Classical mechanics principles in quantum and statistical mechanics

- $\delta S = 0$: $S = \text{action}$

"Hamilton's principle"

(classical mech.) = ($\hbar \rightarrow 0$ limit of
quantum mech.)

- $\Delta S = n\hbar$: $S = \text{action-variable}$
 $= I$

classical harmonic oscillator :

$$H = \omega I$$

$$\Delta I = n\hbar \Rightarrow \Delta E = n\hbar\omega$$

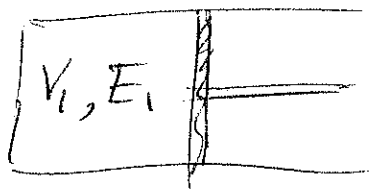
• $\Delta S = 0$ $S = \text{entropy}$

$$= \log(\text{vol. \# microstates})$$

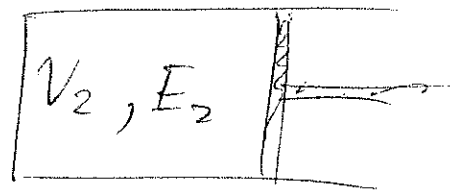
$$= \log(\text{vol. of phase space})$$

adiabatic ($\Delta S = 0$) process preserves volume in phase space

adiabatic expansion of gas:



before



after

monoatomic gas, N atoms

$$E = \sum_{i=1}^{3N} \frac{p_i^2}{2m}$$

\Rightarrow

$$\sum_{i=1}^{3N} p_i^2 = R_p^2 = 2mE$$

sphere in mom. - space

phase space vol. defined by

$$E \pm \Delta E :$$

$$\propto V^N \left(R^{3N}(E+\Delta E) - R^{3N}(E) \right)$$

$$\propto \Delta E \frac{d}{dE} \left(E^{3N/2} \right)$$

$$(\text{phase-space vol.}) \propto V^N E^{3N/2 - 1}$$

$$V E^{3/2} \sim \text{const.}$$

$$E \propto \frac{1}{V^{2/3}}$$

$$P = -\frac{dE}{dV} \propto \frac{1}{V^{5/3}} \quad (\gamma = 5/3)$$