Classical mechanics principles in quantum and statistical mechanic

- $\delta S = 0$: $S =$ action

  "Hamilton's principle"

  (classical mech.) = ($\hbar \to 0$ limit of quantum mech)

- $\Delta S = n\hbar$: $S =$ action-variable $= I$

classical harmonic oscillator :

$$H = \omega I$$

$$\Delta I = n\hbar \implies \Delta E = n\hbar\omega$$

• $\Delta S = 0$ $\quad S$ = entropy

$= \log($~~Vol.~~ # micro-states$)$

$= \log($vol. of phase space$)$

adiabatic ($\Delta S = 0$) process preserves volume in phase space

adiabatic expansion of gas:

before $\qquad$ after

monoatomic gas, N atoms

$$E = \sum_{i=1}^{3N} \frac{p_i^2}{2m} \implies \sum_{i=1}^{3N} p_i^2 = R_p^2 = 2mE$$

sphere in mom.-space

phase space vol. defined by
E & ΔE :

$$\propto V^N \underbrace{\left(R^{3N}(E+\Delta E) - R^{3N}(E) \right)}_{\propto \Delta E \frac{d}{dE}\left(E^{3N/2}\right)}$$

$$(\text{phase-space vol.}) \propto V^N E^{3N/2 - 1}$$

$$V E^{3/2} \sim \text{const.}$$

$$E \propto \frac{1}{V^{2/3}}$$

$$p = -\frac{dE}{dV} \propto 1/V^{5/3} \quad (\gamma = 5/3)$$