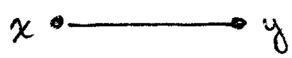
THE BIG IDEA: FREE FIELD THEORY IS NICE, BUT NOTHING LIKE THE REAL WORLD.

THE BIG RESULT OF FREE [SCALAR] FIELD THEORY: FEYNMAN PROP.
$\Rightarrow$ NOW YOU CAN CALCULATE THE PROBABILITY THAT A PARTICLE GOES FROM $x^\mu \to y^\mu$.

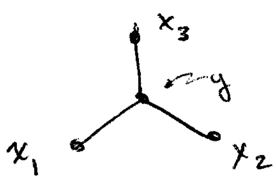
$\rightarrow$ NICE: $\underbrace{\text{CAUSAL, UNITARY}}_{\text{BY CONSTRUCTION}}$ ... EXPLICITLY SOLVABLE!
$\downarrow$
WE HAVE SOLVED THE QUADRATIC PART OF THE ACTION:
$\phi \frac{1}{2}(\partial^2 - M^2)\phi$
SIGNIFICANCE OF QUADRATIC PART EASIER TO SEE IN PATH INTEGRAL FORMALISM.



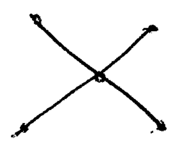
REAL WORLD: PARTICLES INTERACT!

WHAT IS AN INTERACTION?

eg:



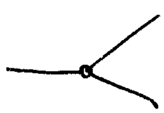
$\sim \phi^3$



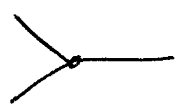
$\sim \phi^4$ $(\sim (a+a^\dagger)^4)$

these are the basic building blocks of feynman diagrams

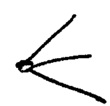
HOW TO READ THIS: $(\xrightarrow{t})$



DECAY INTO 2 PARTICLES



"FUSION" INTO A SINGLE PARTICLE (ANNIHILATION)



SPONTANEOUS GENERATION?!

$\Rightarrow$ SO WE'LL HAVE TO FIGURE OUT HOW THESE THINGS ARE RELATED & HOW SOME OF THEM ARE NOT ALLOWED.

<u>BUT</u>: THIS WEEK WE FOCUSED ON A MORE PRESSING QUESTION:

HOW DO STATES TIME EVOLVE W/RT THIS <u>NEW</u> INTERACTING HAMILTONIAN?

... ARE OUR FREE-FIELD CALCULATIONS MEANINGFUL AT ALL?

ASSUMPTIONS ABOUT THE "QFT I UNIVERSE"

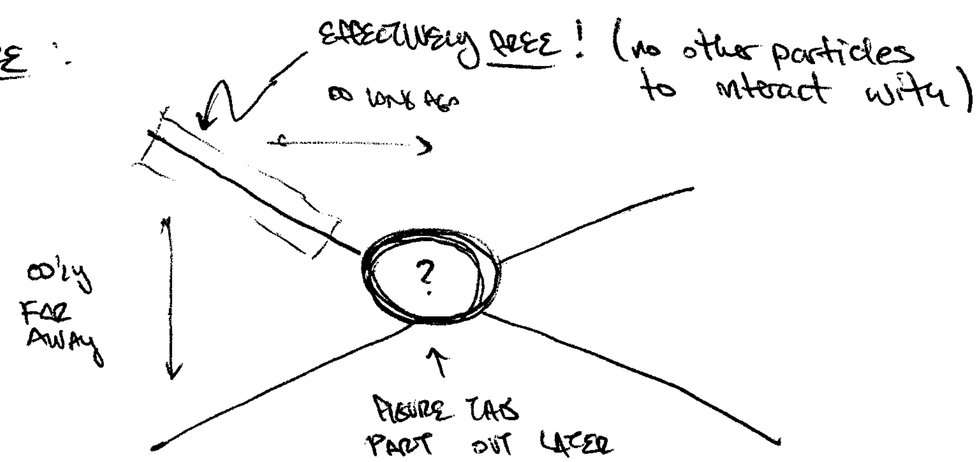
- SPACETIME IS [EFFECTIVELY] INFINITE
  asymptotic states come from $t = \pm\infty$, $|\vec{x}| = \infty$
- [for each amplitude] UNIVERSE IS EMPTY & COLD
  - TEMP = 0, no thermal fluctuations

    ↑ BUT ALLOW <u>QUANTUM</u> FLUCTUATIONS, WHICH ARE ALMOST THE SAME THING
  - NO OTHER PARTICLES AROUND
    ⇒ WE WILL DO SOME CLASSICAL SOURCE CALCULATIONS

SO WHAT TO MAKE OF THESE EXTERNAL STATE PARTICLES THAT ~~ARE~~ COME FROM INFINITELY FAR AWAY, INTERACT W/ EACH OTHER, AND THEN GO THEIR SEPARATE WAYS BACK TO INFINITY?

note: THIS IS A GOOD APPROXIMATION FOR THE REAL WORLD! THE EXTENT OF A POINTLIKE INTERACTION IS SO SMALL THAT MACROSCOPIC DISTANCES (eg LHC) ARE EFFECTIVELY ∞.

<u>PICTURE</u>:



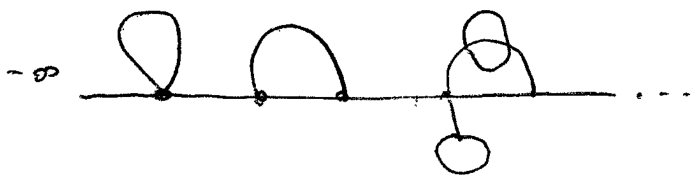
IF THESE STATES ARE FREE, WE CAN ALLOW THEM TO EVOLVE ACCORDING TO THE FREE HAMILTONIAN.

SOUNDS GOOD? NO! IT SHOULD SOUND LIKE BULL SHIT!

WHY? WE ALLOW VERTICES LIKE 

THEN MY EXTERNAL STATE DOESN'T DO THIS:

IT DOES SOME CRAZY STUFF LIKE THIS:



AND, TO MAKE IT WORSE, EACH ONE OF THESE "DRESSED" LINES WE CAN DRAW IS ONE POSSIBLE STATE

THIS SHOULD MAKE YOU FEEL BAD.

BUT EVERYTHING IS OK... YOU'LL JUST HAVE TO WAIT A FEW WEEKS.

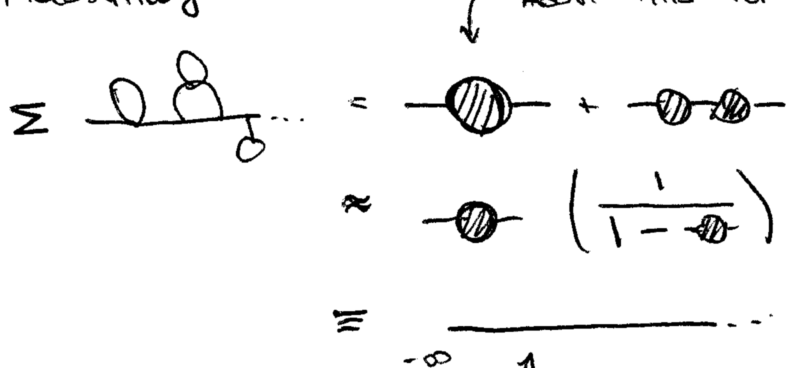
THE MAIN IDEA: ~~[illegible]~~ THESE EFFECTS CAN ALL BE RESUMMED & ACCOUNTED FOR BY SHIFTING PARAMETERS IN OUR THEORY →

1. THE OVERALL FIELD SCALE
2. THE PARTICLE MASS

~~SO THAT THE~~

PICTORIALLY:

"1PI CONNECTED," BUT DON'T WORRY ABOUT THIS FOR NOW.



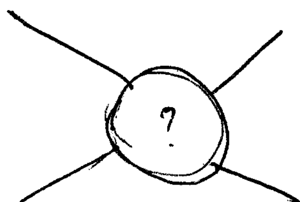
DEFINE A NEW LINE WHERE ALL SELF INTERACTIONS ARE ACCOUNTED FOR! NOT OBVIOUS, BUT YOU CAN DO THIS.

SO: REALLY, WE'RE ASKING YOU TO TAKE ON FAITH THAT WE CAN TALK ABOUT FREE EXTERNAL STATES @ ASYMPTOTIC ∞.

YOU'LL SEE LATER IN THIS COURSE THAT THIS IS COMPLETELY JUSTIFIED.

ALL OF THIS IS TO MOTIVATE ALL THE TIME WE SPENT ON THE INTERACTION PICTURE.

VERY TECHNICAL DISCUSSION
VERY HARD @ 8:30 am.



EXT STATES ARE EFFECTIVELY FREE FIELDS (BUT NOT OBVIOUSLY SO!)

NOW WTF.

FURTHER ASSUMPTION: WEAK COUPLING

COLLOQUIALLY: the force is weak, we can approximate w/ few point interactions

technically: TAYLOR EXPANSION IN SOME SMALL COUPLING CONSTANT.

WHAT IS A COUPLING CONSTANT?

eg. $\mathcal{L} \supset \lambda \phi^4$ — we'll taylor expand in these coefficients.

WHAT DOES THIS # MEAN?

$L = T - V$, so λ TELLS US ABOUT THE ENERGY COST OF INTERACTING w/ 3 OTHER ϕs.

NOTE: λ SMALL $\rightarrow \ll \mathcal{O}(1)$

WHAT ABOUT $\mathcal{L} \supset g\phi^3$. WHAT DOES "$g$ = SMALL" MEAN?

$\hookrightarrow$ g IS NOT DIMENSIONLESS! (ANOTHER IMPORTANT "BIG PICTURE" THING TO NOTICE)

SO BASICALLY WE'RE SAYING

$$\mathcal{L} = \underbrace{\mathcal{L}_0}_{\text{FREE.}} + \sum_{n=3}^{\infty} g_n \phi^n$$

SMALL IN SOME SENSE } IN ~~WHAT~~ WHAT SENSE IS IMPORTANT

eg. COLLISIONS @ LHC VS. LOW E DM

POTENTIALLY ∞ NUMBER of TERMS!
HOW DO WE KNOW THAT WE ONLY NEED A FEW?

↳ think about this w/ dimensional analysis.

must introduce some scale Λ to make g_n dimensionless.

WE TREAT $\mathcal{L}_0$ EXACTLY (WE'VE DONE THIS)

↳ PERTURB ABOUT $g_n = 0$.
[DOESN'T WORK FOR, eg, LOW ENERGY QCD!]

FOR SIMPLICITY, SET $t_0 = 0$.

HEISENBERG PICTURE: $\phi(t,\vec{x}) = e^{iH(t-t_0)}\,\phi(t_0,\vec{x})\,e^{-iH(t-t_0)}$

FIELD IS AN OPERATOR (SCHRÖDINGER PIC)

THE TIME EVOLUTION IS ~~DOMINATED. IT'S THE~~ DOMINATED (BY ASSUMPTION) BY THE FREE EVOLUTION

$$\phi(x) \approx \phi_I(x) \equiv e^{iH_0 t}\,\phi(x_0)\,e^{-iH_0 t}$$

$(0,\vec{x})$

IN WORDS: TO GOOD APPROXIMATION, NO INTERACTIONS.

THE TIME EVOLUTION of ϕ_I IS COMPLETELY UNDERSTOOD (MODULO THE STATEMENTS ABOUT LOOPS)

$$\phi_I = \int d^3k \frac{1}{\sqrt{2E_k}} \left(a_k e^{-ik\cdot x} + a^\dagger_k e^{ik\cdot x}\right) \Big|_{x^0 = t}$$

$$\phi(x) = \phi_I(x) + \mathcal{O}(\lambda) \cdots$$

CLEARLY THE ZEROTH O APPROX IS NO GOOD FOR SCATTERING.
NEED TO DEVELOP SYSTEMATIC EXPANSION.

$$\phi(x) = e^{iHt}\,\underbrace{e^{-iH_0 t}\,\phi_I(x)\,e^{iH_0 t}}_{\text{UNDO FREE TIME EVOLUTION}}\,e^{-iHt}$$

THEN REDO IT, BUT NOW W/ FULL INTERACTION HAMILT.

stupidly obvious formula, don't get confused!

CALL THIS $U^{+}(t)$

SCHRÖDINGER PICTURE

$$i\,\frac{d|\psi\rangle_S}{dt} = H_S|\psi\rangle_S$$

$$i\,\frac{d}{dt}\left(e^{-iH_0 t}|\psi\rangle_I\right) = (H_0 + H_{int})_S\, e^{-iH_0 t}|\psi\rangle_I$$

$$(\cancel{H_0} + i\tfrac{d}{dt})|\psi\rangle_I = (\cancel{H_0} + H_{int})_S\, e^{-iH_0 t}|\psi\rangle_I$$

$e^{-iH_0 t}$

$$\Rightarrow i\frac{d}{dt}|\psi\rangle_I = \underbrace{e^{iH_0 t}(H_{int})_S\, e^{-iH_0 t}}_{H_I}|\psi\rangle_I$$

H_I INT. HAMILTONIAN IN INT. PICTURE.

SOLUTION: $|\psi(t)\rangle_I = U(t)\,|\psi(0)\rangle_I$

SAME $U(t)$ AS ABOVE.

THEN: $$i \frac{d}{dt} U(t) |\psi(0)\rangle_I = H_I(t)\, U(t)\, |\psi(0)\rangle_I \quad \star$$

naive: $$U(t) \stackrel{?}{=} e^{-i\int_0^t H_I(t')\,dt'}$$ (from usual ODE solution for c-#s)

WRONG! $$\text{RHS} = 1 - i\left(\int\cdots\right) + \frac{(-i)^2}{2}\left[\underline{\underline{\left(\int_0^t H_I(t')\,dt'\right) H_I(t)}} + H_I(t)\left(\int\cdots\right)\right]$$

UNDERLINED TERM: ~~NOT RIGHT~~! H_I SHOWS UP ON WRONG SIDE! CANNOT GIVE ($\star$) SINCE $[H_I(t), H_I(t')] \neq 0$!

REAL SOLUTION: DYSON'S FORMULA

$$\boxed{U(t) = T e^{-i\int_0^t H_I(t')\,dt'}}$$

$$\begin{aligned} U(t) = 1 &- i\int_0^t dt'\, H_I(t') \\ &- \frac{1}{2}\left[\int_0^t dt' \int_{t'}^{t} dt''\, H_I(t'')\, H_I(t') + \int_0^t dt \int_0^{t'} dt''\, H_I(t')\, H_I(t'')\right] = \int_0^t dt' \int_0^{t'} dt''\, H_I'\, H_I'' \\ &+ \cdots \end{aligned}$$

WHY: UNDER T ALL OPERATORS COMMUTE (ORDER FIXED BY T)

$$i\frac{d}{dt} T e^{-i\int dt'\, H_I'} = T\left[H_I(t)\, e^{-i\int dt'\, H_I'}\right]$$

CAN PULL THIS OUT SINCE t IS UPPER LIMIT OF INTEGRAL.

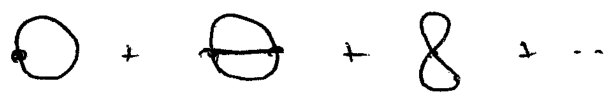
REMARK: PESKIN P. 86-87

DISCUSSION of THE VACUUM IN AN INTERACTING THEORY

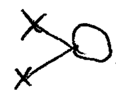
FREE THY: $|0\rangle$ IS THE VACUUM

INTERACTING THY: HAMILTONIAN IS DIFFERENT!
VACUUM IS A DIFFERENT STATE
W/RT FREE THEORY FOCK STATES!

PICTORIALLY, VACUUM INCLUDES:



OR EVEN THINGS LIKE



← HIGGS

PREVIEW of FEYNMAN DIAGRAMS

WICK'S THM

$$T[\phi_x \phi_y] = N[\phi_x \phi_y + \contraction{}{\phi}{_x}{\phi}\phi_x \phi_y]$$

↑ NORMAL ORDER

↑ WICK CONTRACTION (FEYNMAN PROPAGATOR)

→ & GENERALIZ TO HIGHER POINT CORRELATORS.

SO WHAT? $|\text{state}\rangle \to U(t)|\text{state}\rangle$

$\langle \text{out}| U^\dagger U |\text{in}\rangle$

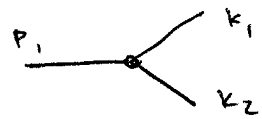
$\sim T e^{i\int dt' H_I}$

↑ TAYLOR EXPAND

eg to LINEAR ORDER: $g\phi^3$ ← @ SOME SPACETIME POINT.

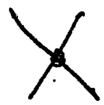
SO, HEURISTICALLY:

$$\langle \vec{k}_1, \vec{k}_2 | \cdots + g\phi^3 + \cdots | \vec{p}_1 \rangle \sim g$$



WHAT ABOUT 2-2 SCATTERING?

$$\langle \vec{k}_1, \vec{k}_2 | \cdots + \lambda\phi^4 + \cdots + \tfrac{1}{2}(g\phi^3)^2 + \cdots | \vec{p}_1, \vec{p}_2 \rangle$$



WHAT TO MAKE OF THIS?

$$\sim (a+a^\dagger)^6$$

$$\sim \underset{\text{HITS FINAL STATE}}{(a^\dagger)^2} \; \underbrace{(a+a^\dagger)(a+a^\dagger)}_{\text{HITS EACH OTHER}} \; \underset{\text{HITS INIT STATE}}{a^2}$$

<u>THIS</u> IS A WICK CONTRACTION.

HIGHER ORDER

$$g^2 \langle \vec{k}_1 \vec{k}_2 | \; \underbrace{\phi\phi\phi}_{x} \; \underbrace{\phi\phi\phi}_{y} \; | \vec{p}_1 \vec{p}_2 \rangle$$

PICTURE:

WE'VE BEEN VERY SLOPPY – BUT YOU'LL SEE THE GORY DETAILS IN LECTURE @ 8:30 am.

↳ MUST KEEP TRACK OF
- MOMENTUM CONSERVATION
- PERMUTATIONS
- COMBINATORIAL FACTORS

FINAL REMARKS ABOUT SCALING

INTERNAL SYM: $\delta\mathcal{L} = 0$
TRANSLATION + LORENTZ: $\delta\mathcal{L} = \partial_\mu f^\mu$

SCALING: $d^4x\, \delta\mathcal{L} = d^4x\, \partial_\mu f^\mu$, but $\delta\mathcal{L}$ NOT NEC $\partial_\mu f^\mu$

SPACETIME TRANSL:

$$\delta\mathcal{L} = \partial_\mu f_\mu$$

$$\hookrightarrow = \begin{cases} (\partial_\mu \mathcal{L})\, \delta x^\mu \\ \frac{\delta\mathcal{L}}{\delta\phi}\delta\phi + \frac{\delta\mathcal{L}}{\delta\partial\phi}\delta\partial\phi \longrightarrow \partial_\mu\left(\frac{\delta\mathcal{L}}{\delta\partial_\mu\phi}\delta\phi\right) \end{cases}$$

$$\delta\mathcal{L} = (\partial_\mu \mathcal{L})\,\delta x^\mu = \partial_\mu\left(\frac{\delta\mathcal{L}}{\delta\partial_\mu\phi}\delta\phi\right)$$

neither $\mathcal{L}\, dx^\mu$ nor $\uparrow$ ARE CONSERVED CURRENTS
→ NOT DIVERGENCE-FREE

BUT TRIVIALLY

$$\delta\mathcal{L} - \delta\mathcal{L} = \partial_\mu\Big[\underbrace{\mathcal{L}\,\delta x^\mu - \frac{\delta\mathcal{L}}{\delta\partial_\mu\phi}\delta\phi}_{\equiv j^\mu}\Big] = 0$$

LET $\delta x^\mu = \delta^\mu{}_\nu\, \varepsilon^\nu$, $\delta\phi = (\partial_\nu\phi)\,\varepsilon^\nu$ } GET $T^\mu{}_\nu$

HOW IS SCALING DIFFERENT?

$$\delta(d^4x\, \mathcal{L}) = d^4x\, \partial_\mu f^\mu$$

$$\hookrightarrow \begin{cases} (\partial_\mu \delta x^\mu)\mathcal{L} + (\partial_\mu\mathcal{L})\,\delta x^\mu = \partial_\mu(\mathcal{L}\,\delta x^\mu) \\ \partial_\mu\left(\frac{\delta\mathcal{L}}{\delta\partial_\mu\phi}\delta\phi\right) \end{cases}$$

$$j^\mu = \left(\mathcal{L}\,\delta x^\mu - \frac{\delta\mathcal{L}}{\delta\partial_\mu\phi}\delta\phi\right)$$

OTHER REMARKS

ANTIPARTICLES : $C = \underbrace{PT}_{\text{SPACETIME SYMMETRIES}}$

ANTIHERMITIAN (GIVES DAGGER)

CHARGE CONJUGATION IS KIND OF A SPACETIME SYM!

IN HW: EASY QUESTION : CPT INVARIANCE
IN PREVIOUS HW:

IN NONRELATIVISTIC THEORY,

$$Q \sim \int d^3x\ \phi^*\phi \ \sim \underbrace{\sum a_k^\dagger a_k}_{\text{PARTICLE \# OPERATOR}}$$

BUT IN QFT PARTICLE # NOT CONSERVED b/c of ANTIPARTICLES

$$\phi \sim \int d^4k\ a_k\, e^{ik\cdot x}$$

no antiparticle! (even though ϕ SCALAR!)

NOTE: WE'RE TALKING TECHNICALLY ABOUT ANTIPARTICLES.

NO MUMBO-JUMBO ABOUT PARTICLE + ANTIPARTICLE ANNIHILATE INTO "PURE ENERGY".

DON'T EVEN KNOW WHAT THIS MEANS.

LAST REMARK: HOW ANTIMATTER SAVED THE POINTLIKE ELECTRON.